A Practical Model for Hair Interaction

Connelly Barnes
William Brendel
Nadia Payet

Introduction

- Produce realistic looking hair

- Dynamics of long hair is difficult
  - High number of hair strands
  - Complex physical interactions

Outline

- Hair modeling
- Single hair strand dynamics
- A sparse model for hair-hair interaction
  - Static links
  - Dynamic interactions
- Create dense hair with interpolation
- Hair rendering
- Demos

Hair modeling

- Few hundred strands à guide hair
- Each strand has multiple segments connected by vertices

- Structural elements added
  - Connections between vertices
  - Triangular meshes among guide hair

Hair modeling

- Previous publication

  Modeling realistic virtual hairstyles –Y.Yu

Single hair strand dynamics

- Rigid multi-body chain
- Rotational joint between segments
- à simple articulated body

- Forward dynamics solved using
  - Lagrange's equations
  - Articulated-Body method (linear time)

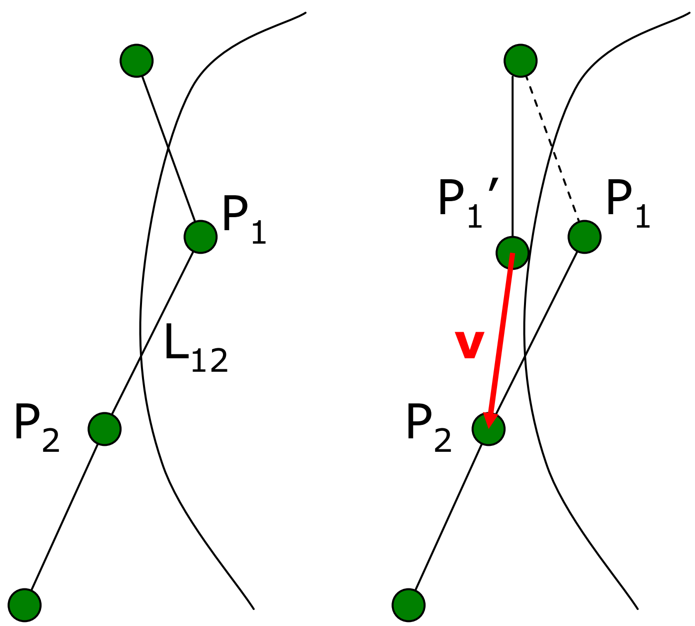
Strand-Object collision

- Hair vertex too close from object
- Set its velocity to the velocity of the object

- Check penetration of hair strand particles with the triangle mesh of the objects

Strand-Object collision

- If penetration occurs
  - We need to move vertex out
  - We also move remaining part of strand



$$p_2' = p_1' + v'$$

$$v' = L_{21}\frac{v}{\|v\|}$$

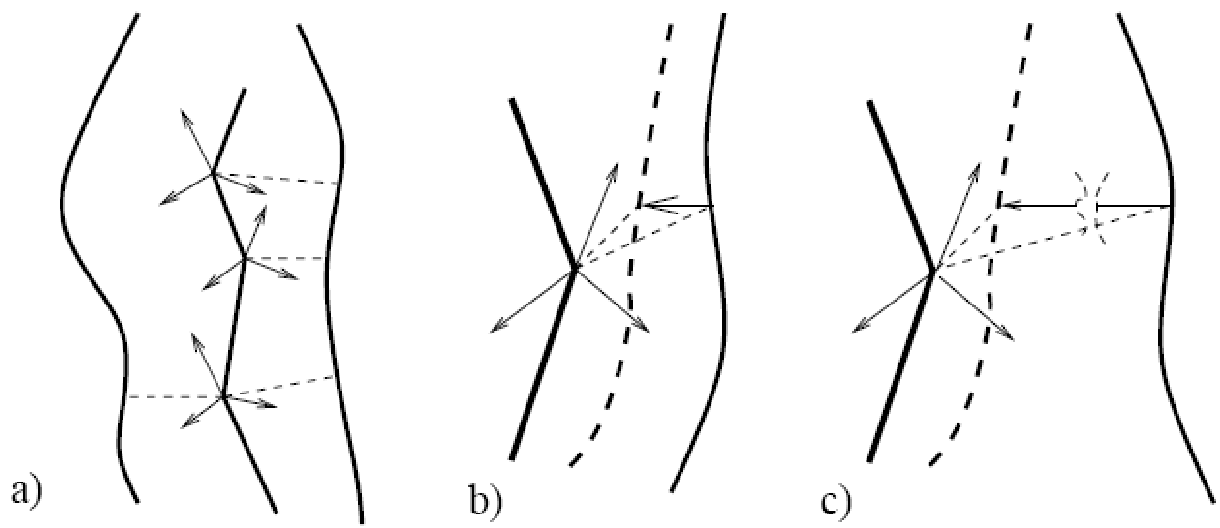
Repeat until we reach tip of strand

Hair-Hair interaction

- Static links
  - For hairs bonding effect
  - Simulate the elastic lateral motion and enable hairstyle recovery
  - Hair à elastically deformable volume
  - Links = breakable connections
  - Links = springs with zero resting length
  - Each segment connected to n closest neighbors

Static Links

$$\mathbf{f}_h = \sum_i \left[k^s_{h,i} |\mathbf{l}_i| - k^d \frac{\mathbf{v}_i \cdot \mathbf{l}_i}{|\mathbf{l}_i|} \right] \frac{\mathbf{l}_i}{|\mathbf{l}_i|}$$



Dynamic interactions

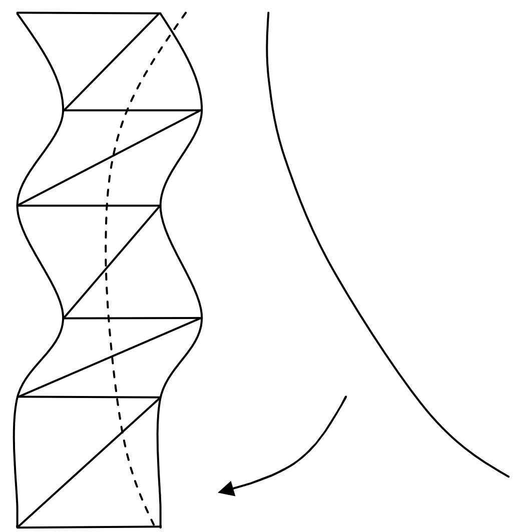
- Two objects in dynamic interactions: hair segments and triangle strips.
- Build triangle strips between guide hairs with adjacent roots.
- Place these in an octree or *k*d-tree for speed.

Dynamic interactions

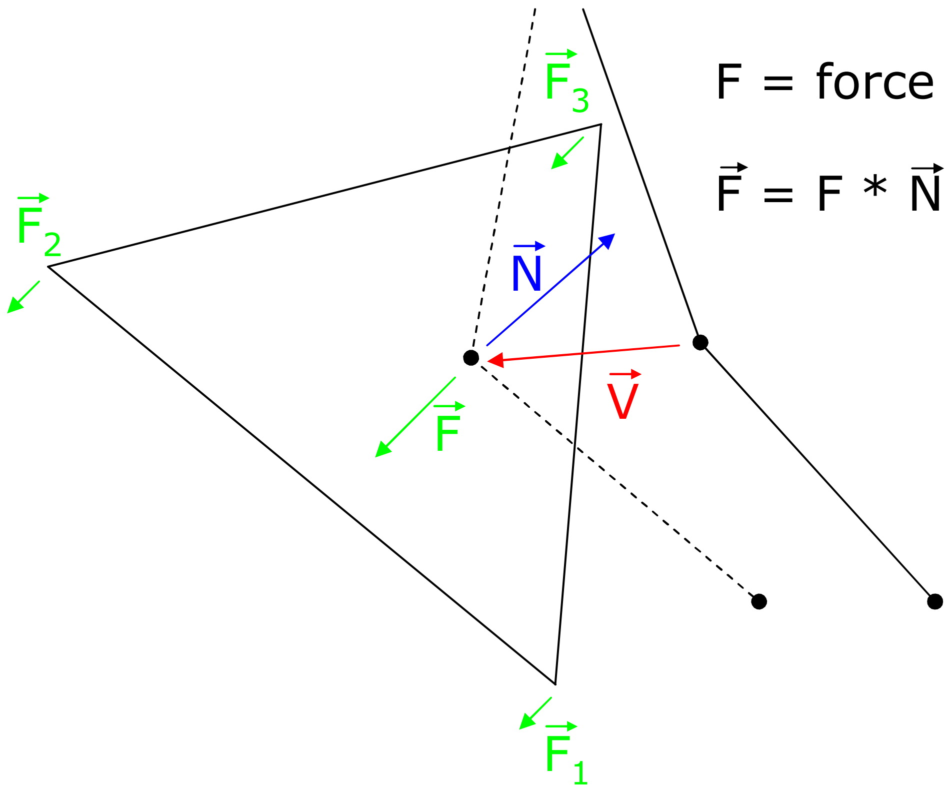
- Detect collisions: hair vertex-triangle face and hair segment-hair segment.
- Collisions occur when the two geometries are less than a threshold distance apart.
- Forces result from collision: spring force and friction. Forces on triangle are distributed to vertices.

Dynamic interactions

- Dynamic interaction
  - Use triangle strip between hairs
  - Allowed when hairs have nearby roots
  - Triangle patches used for collision



Dynamic interactions



$F = \text{force applied} * (\vec{N}.\vec{V})/||\vec{V}||$

$\vec{F} = F * \vec{N}$

Dynamic interactions

- Friction:

$$\mathbf{F}_{fric} = -\mu \mathbf{F}_N \frac{\mathbf{v}_{\text{rel, in plane}}}{\left\| \mathbf{v}_{\text{rel, in plane}} \right\|}$$

- Plane determined by two segments, or face.
- Spring:

Term goes to zero as hairs line up.

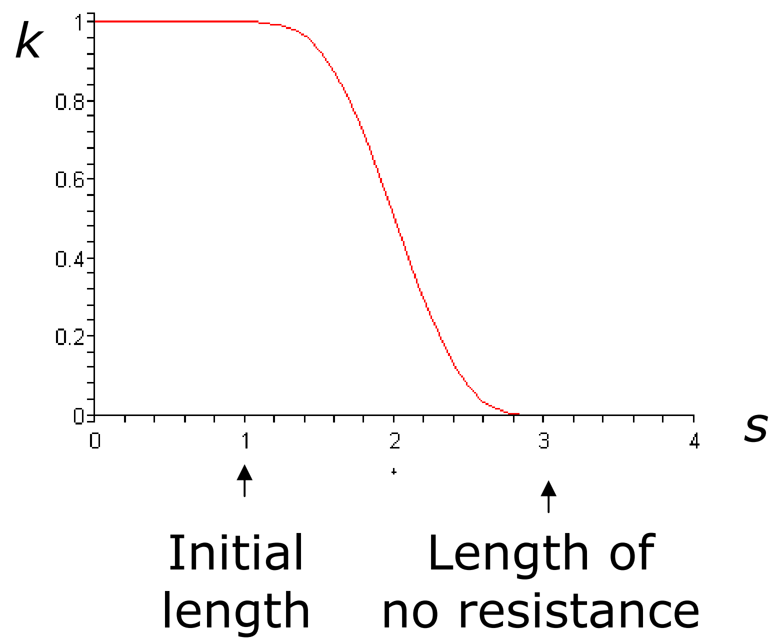
$$\mathbf{F}_s = k d_{\text{a,b}} (1 - \left| \mathbf{T}_{\text{a}} \cdot \mathbf{T}_{\text{b}} \right|)$$

Segment-segment or vertex-face distance

Tangent vectors for hair, computed in triangle by e.g. **u** direction.

Dynamic interactions

- For vertex-face collisions, spring constant is a function of hair density on face.
- One solution: *k* a function of "horizontal" triangle edge length *s*:

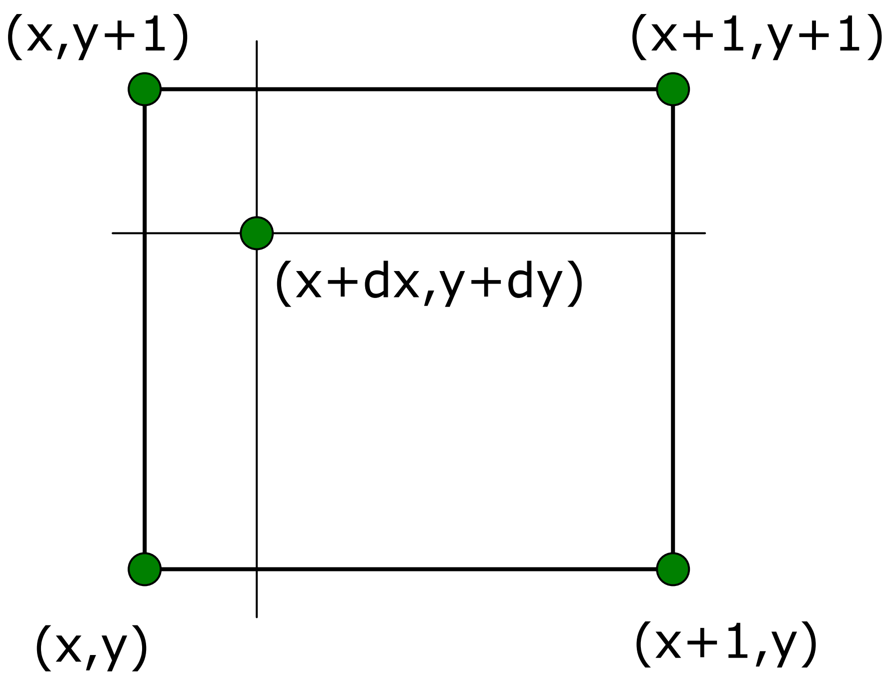


Hair interpolation

- Each hair from sparse model serves as a guide hair
- Remaining hair strands in dense set are interpolated from these guide hairs

Hair interpolation

- Bilinear interpolation between guide hairs

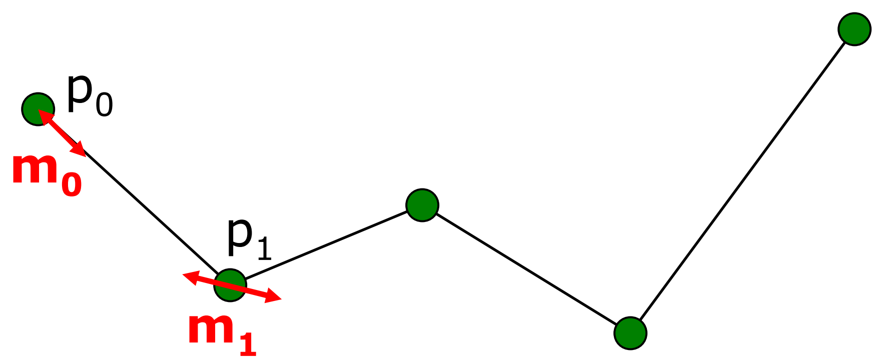


$$
\begin{aligned}
&P(x+dx, y+dy) = \\
&\quad (1-dx)(1-dy)P(x, y+1) \\
&+ (1-dx)dyP(x, y+1) \\
&+ dx(1-dy)P(x, y+1) \\
&+ dxdyP(x, y+1)
\end{aligned}
$$

Hair interpolation

- Hermite Spline interpolation
  - Improve smoothness of strands
  - Only 10 to 15 segments are enough to have a smooth output

$$p(t) = (2t^3 - 3t^2 + 1)p_0 + (t^3 - 2t^2 + t)m_0 + (-2t^3 + 3t^2)p_1 + (t^3 - t^2)m_1$$



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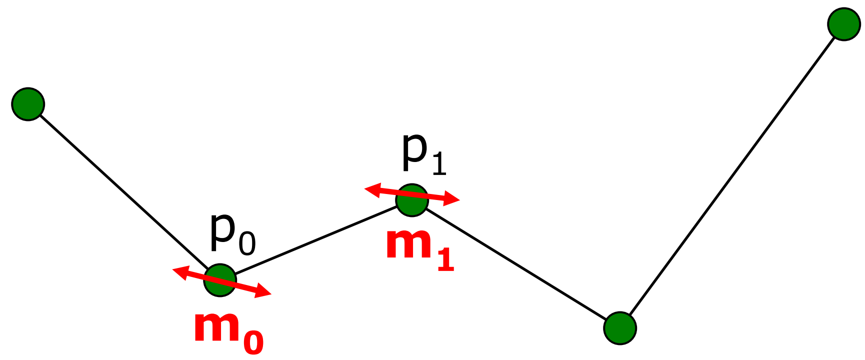

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