Global Parametrization

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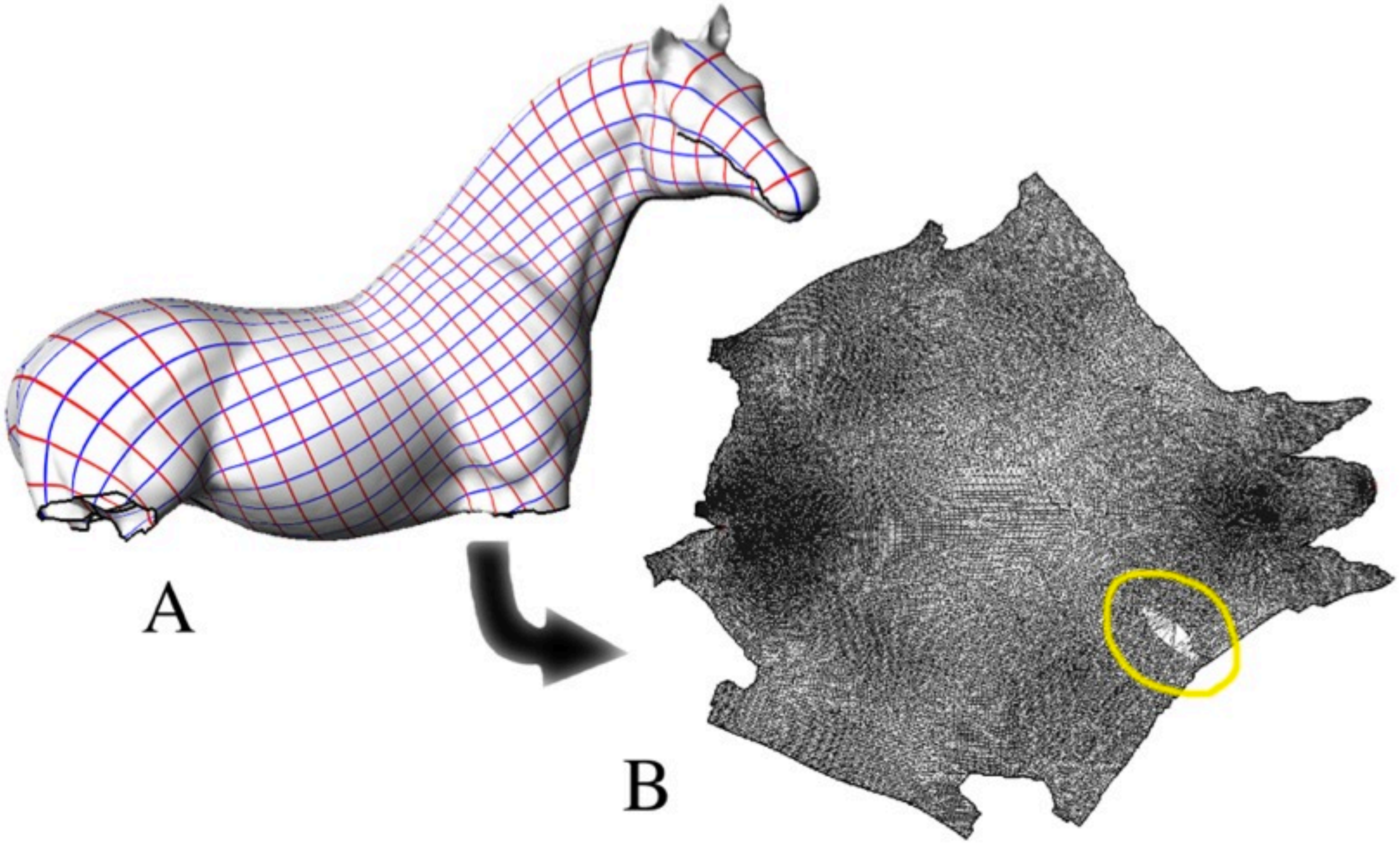
Overview

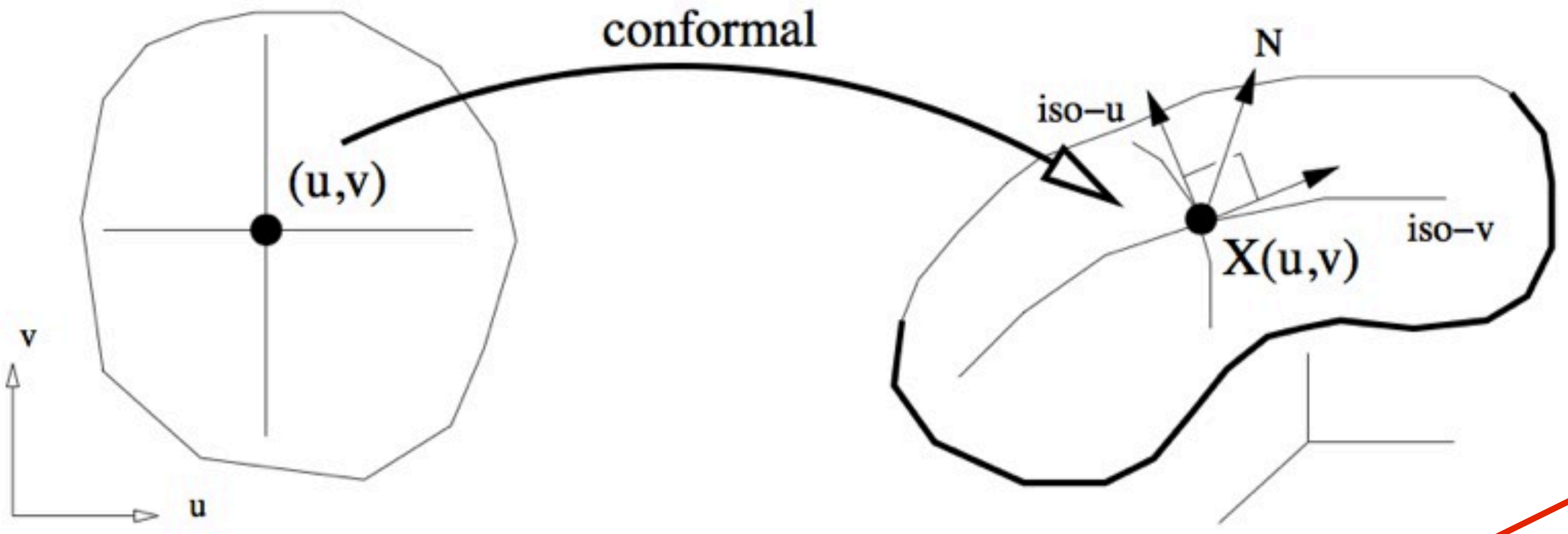
- Boundary-free parametrization
- Global parametrization
- Cross-field aligned parametrization

Bundary-free



Conformal Maps

- In a conformal map the tangent vector to the iso-u and iso-v are orthogonal and have the same length



Complex numbers!

$$N(u,v) \times \frac{\partial \mathcal{X}}{\partial u}(u,v) = \frac{\partial \mathcal{X}}{\partial v}(u,v) \longrightarrow \frac{\partial \mathcal{X}}{\partial u} - i\frac{\partial \mathcal{X}}{\partial v} = 0$$

Conformality on a Triangulation

- The conformality condition can be rewritten in the local frame of every triangle:

$$\left(N(u,v) \times \frac{\partial \mathcal{X}}{\partial u}(u,v) = \frac{\partial \mathcal{X}}{\partial v}(u,v) \right) \longrightarrow \frac{\partial \mathcal{X}}{\partial u} - i\frac{\partial \mathcal{X}}{\partial v} = 0$$

- Using the theorem of the derivatives of the inverse functions:

$$\frac{\partial \mathcal{U}}{\partial x} + i\frac{\partial \mathcal{U}}{\partial y} = 0$$

Least-Square Conformality

- This condition cannot be strictly enforced for every triangle, but we can minimize it:

$$C(T) = \int_T \left| \frac{\partial \mathcal{U}}{\partial x} + i \frac{\partial \mathcal{U}}{\partial y} \right|^2 dA = \left| \frac{\partial \mathcal{U}}{\partial x} + i \frac{\partial \mathcal{U}}{\partial y} \right|^2 A_T$$

- Summed over the triangulation:

$$C(\mathcal{T}) = \sum_{T \in \mathcal{T}} C(T)$$

Computation of LSCM

- Writing the previous equations with respect to the positions of the vertices in the plane result in a simple least-square system:

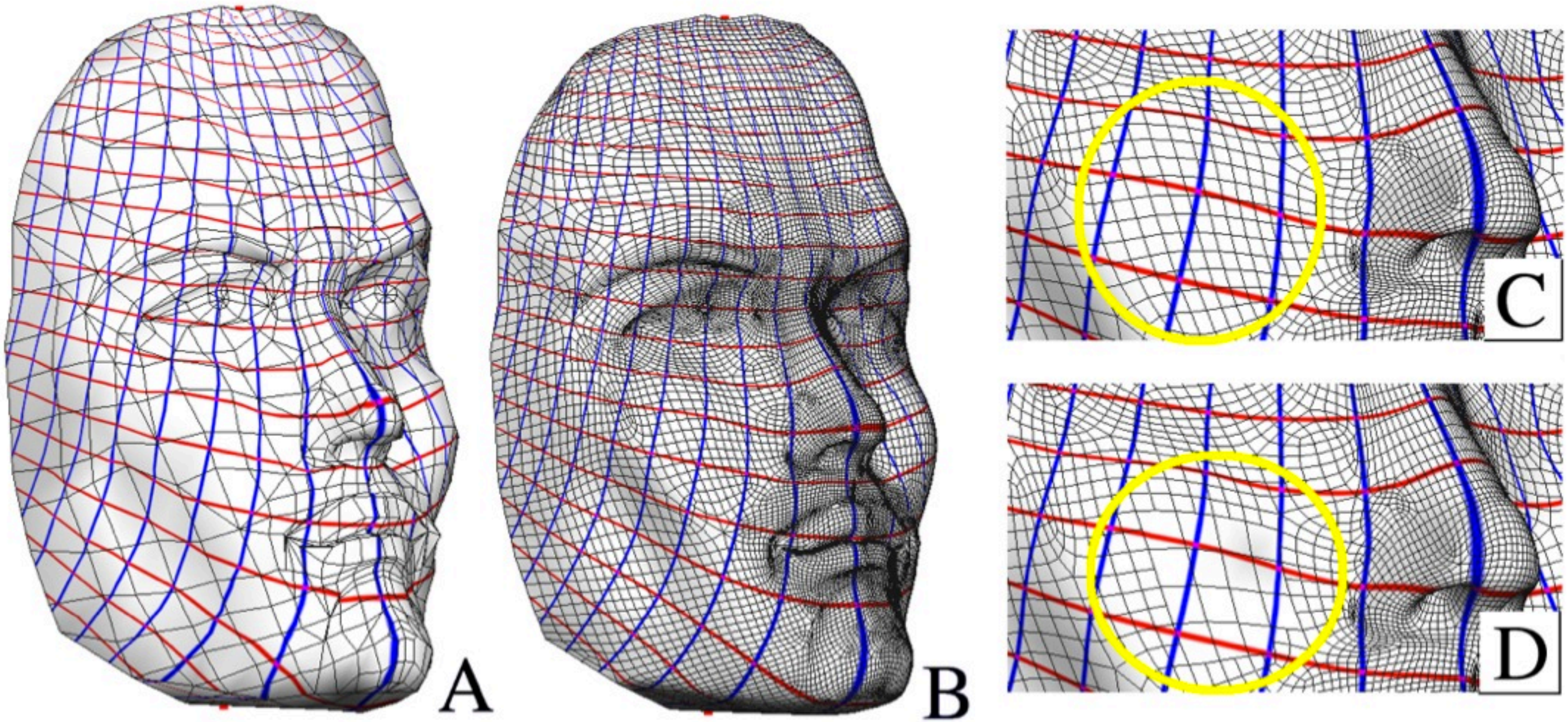
$$C(\mathbf{x}) = \|A\mathbf{x} - \mathbf{b}\|^2$$

- At least two points must be fixed to make **A** full-rank

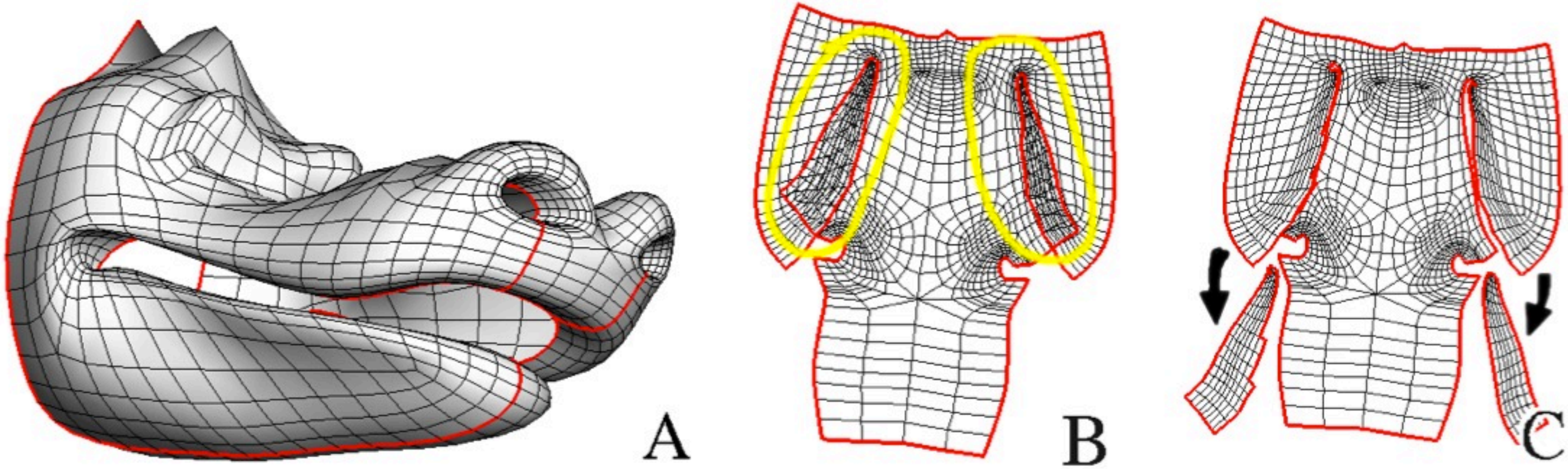
Properties of LSCM

- The solution is unique
- The solution is invariant to similarity in texture space
- The solution is independent of the resolution
- Triangle flips are rare (they only happens with bad triangulations)

Resolution independence



Global overlaps are not prevented



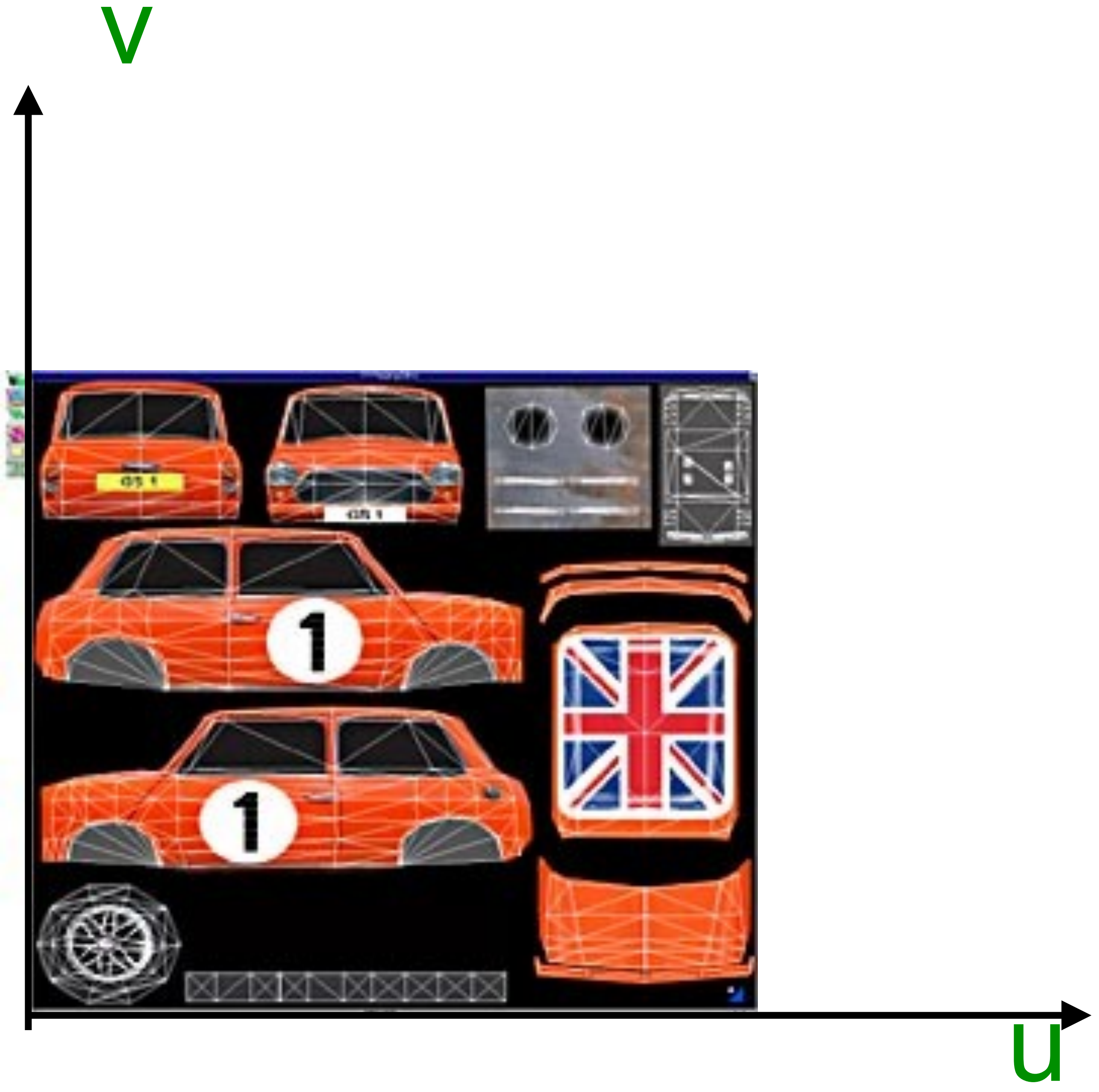
Commercial applications

- Blender implements LSCM
- http://www.blender.org/download/sandbox/lscm-basics/

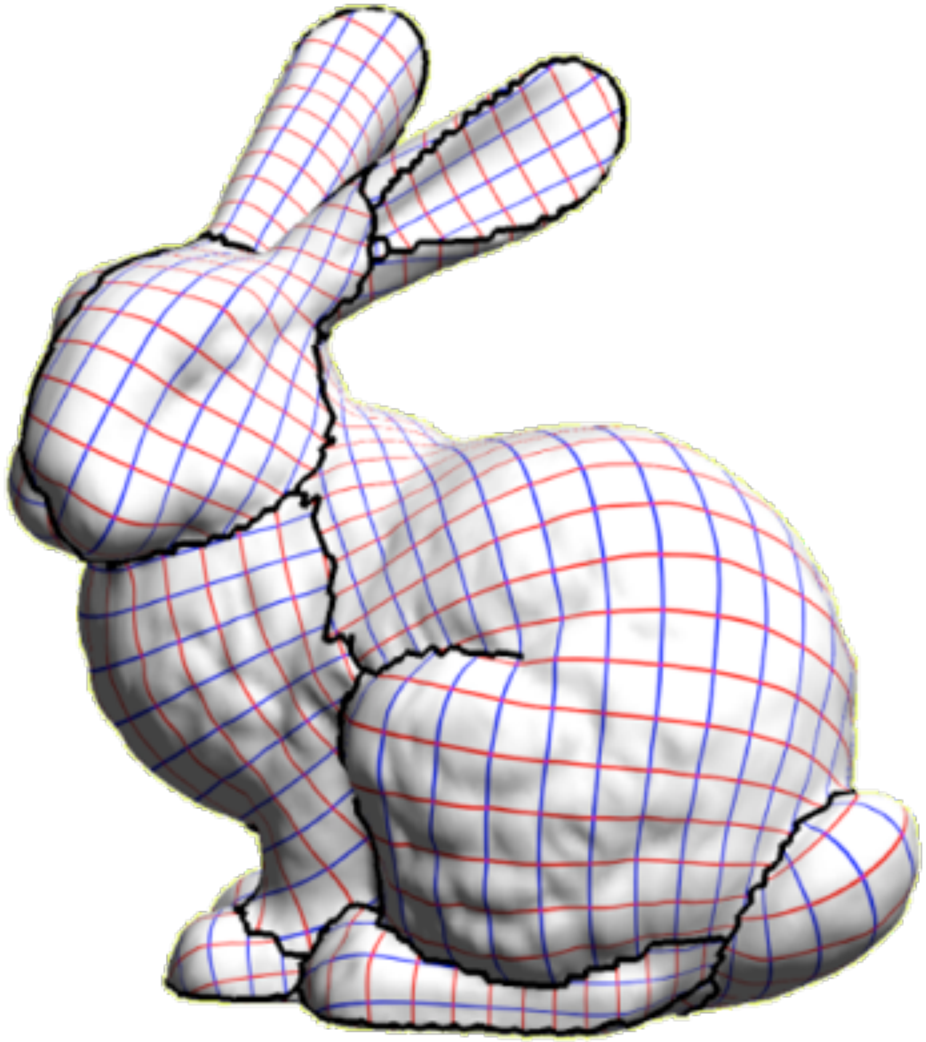
Global parametrization

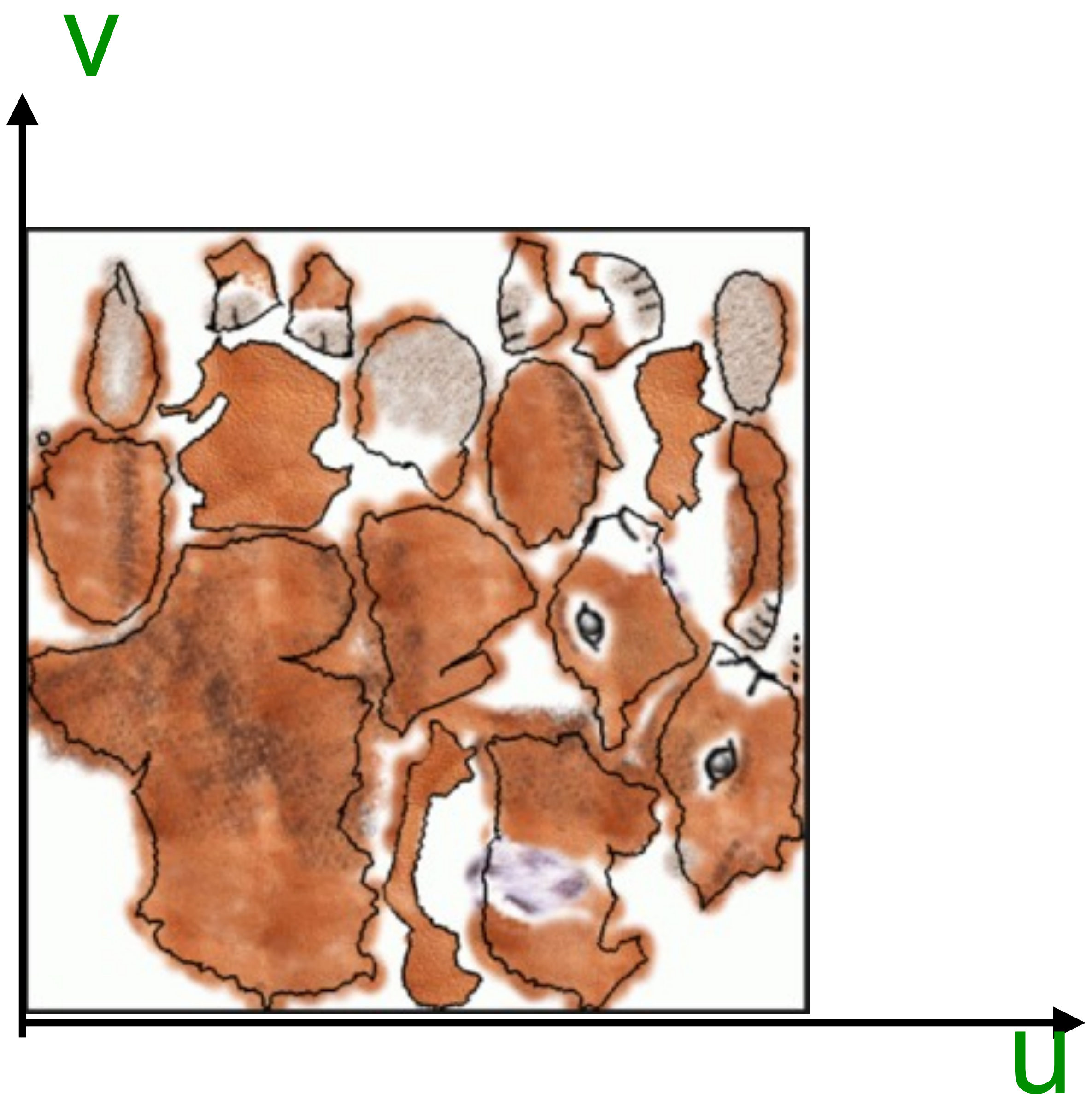
- All the algorithms that we studied till now are able to parametrize a part of a mesh homeomorphic to a disk
- To be able to parametrize arbitrary shapes we “cut” a mesh into parts, and we parametrize every part independently

Manual parametrization

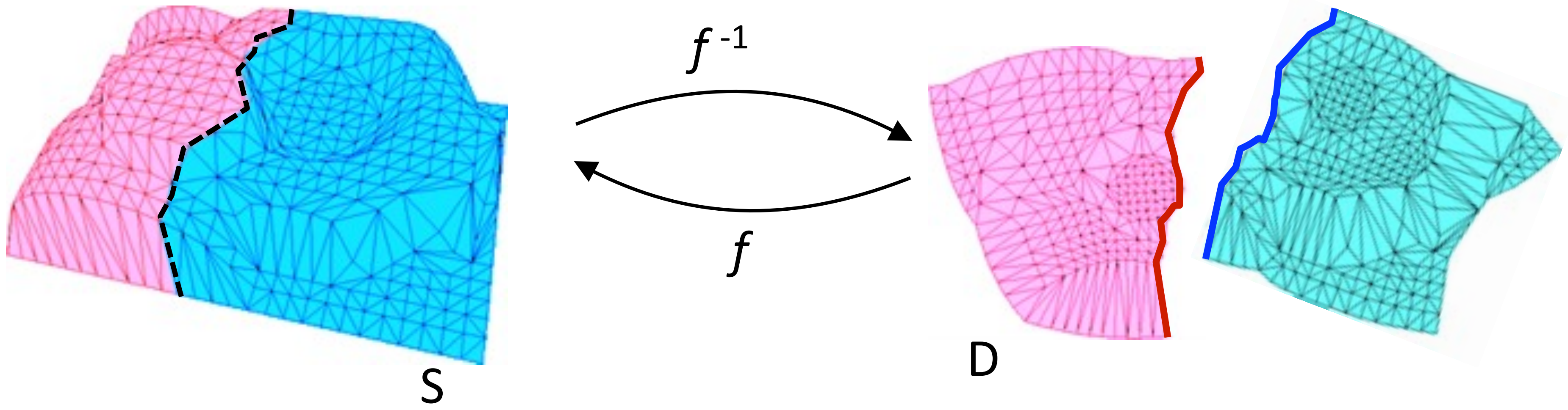


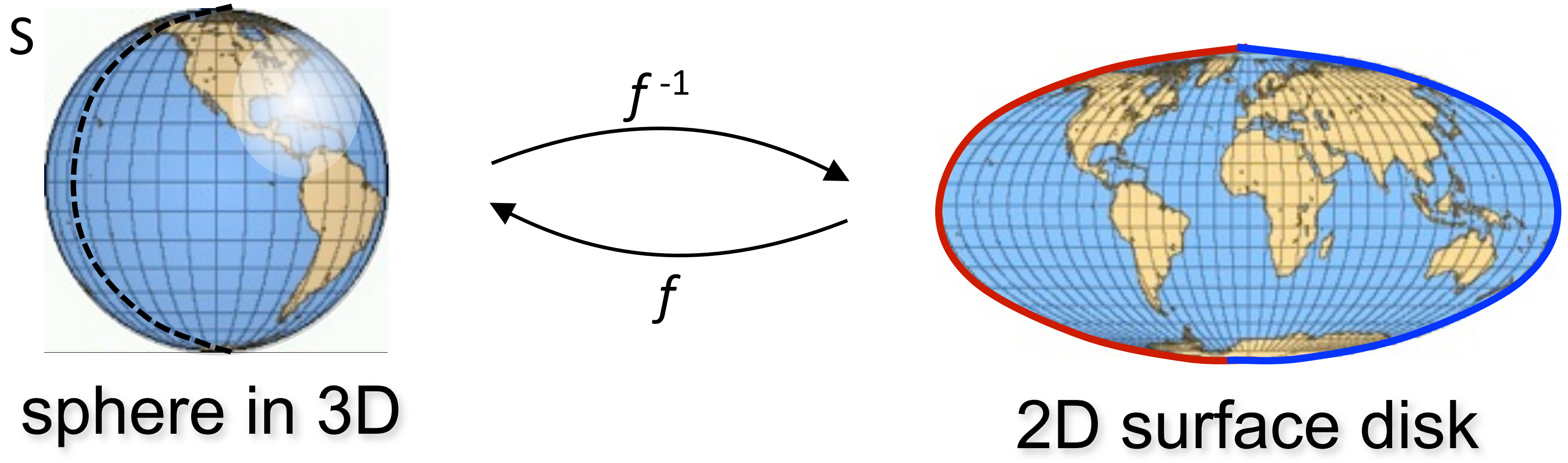
Automatic Parametrization





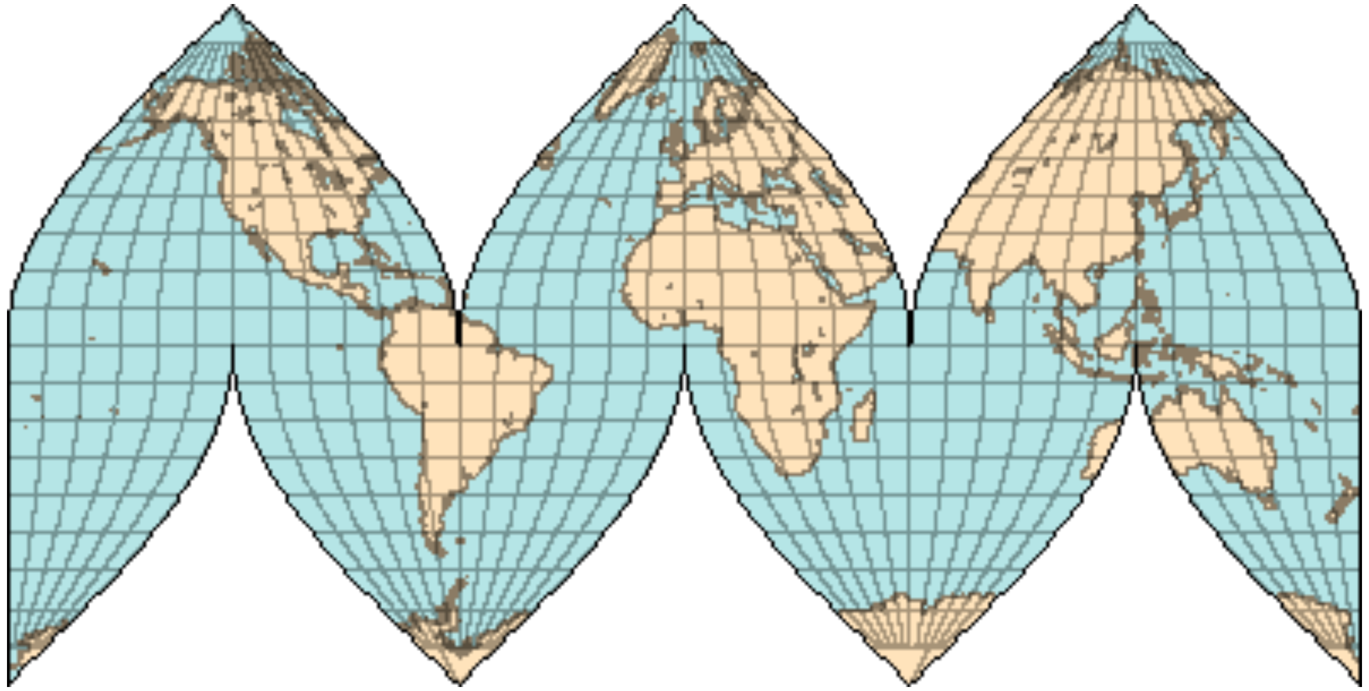
Good = “fewer cuts”?

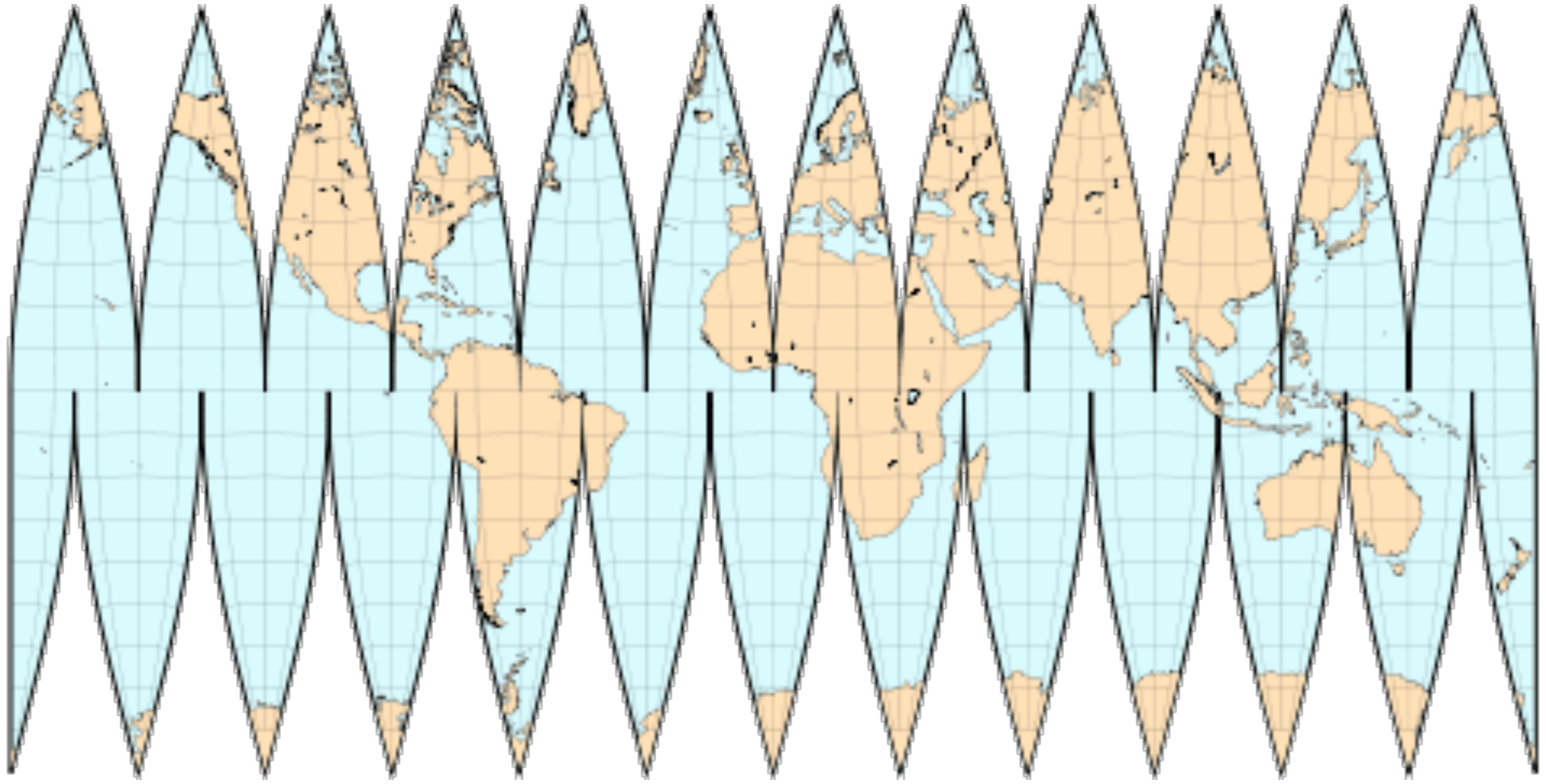




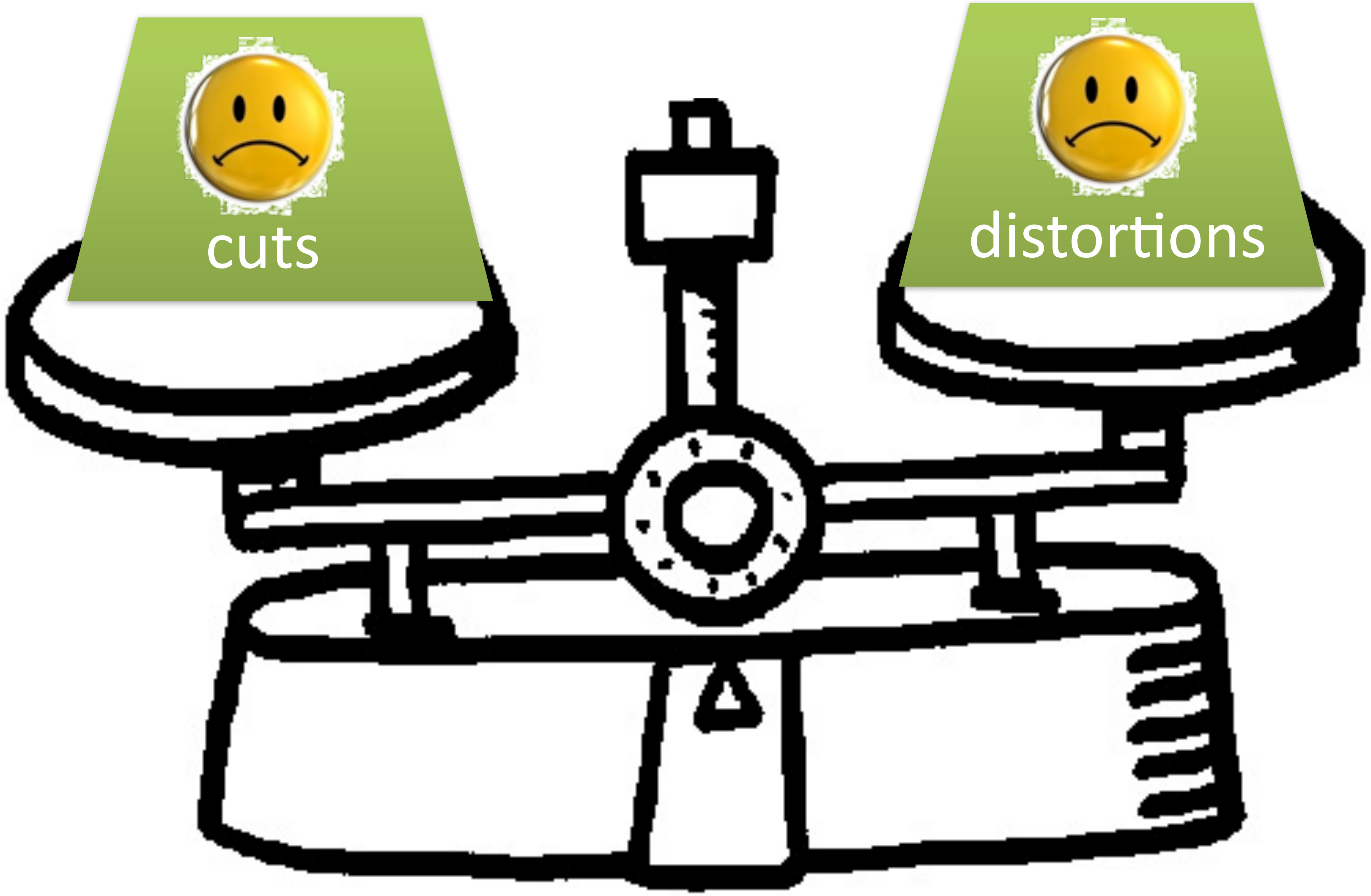
Good = "fewer cuts"?

but… more cuts => less distortion





A difficult balance



How to handle cuts?

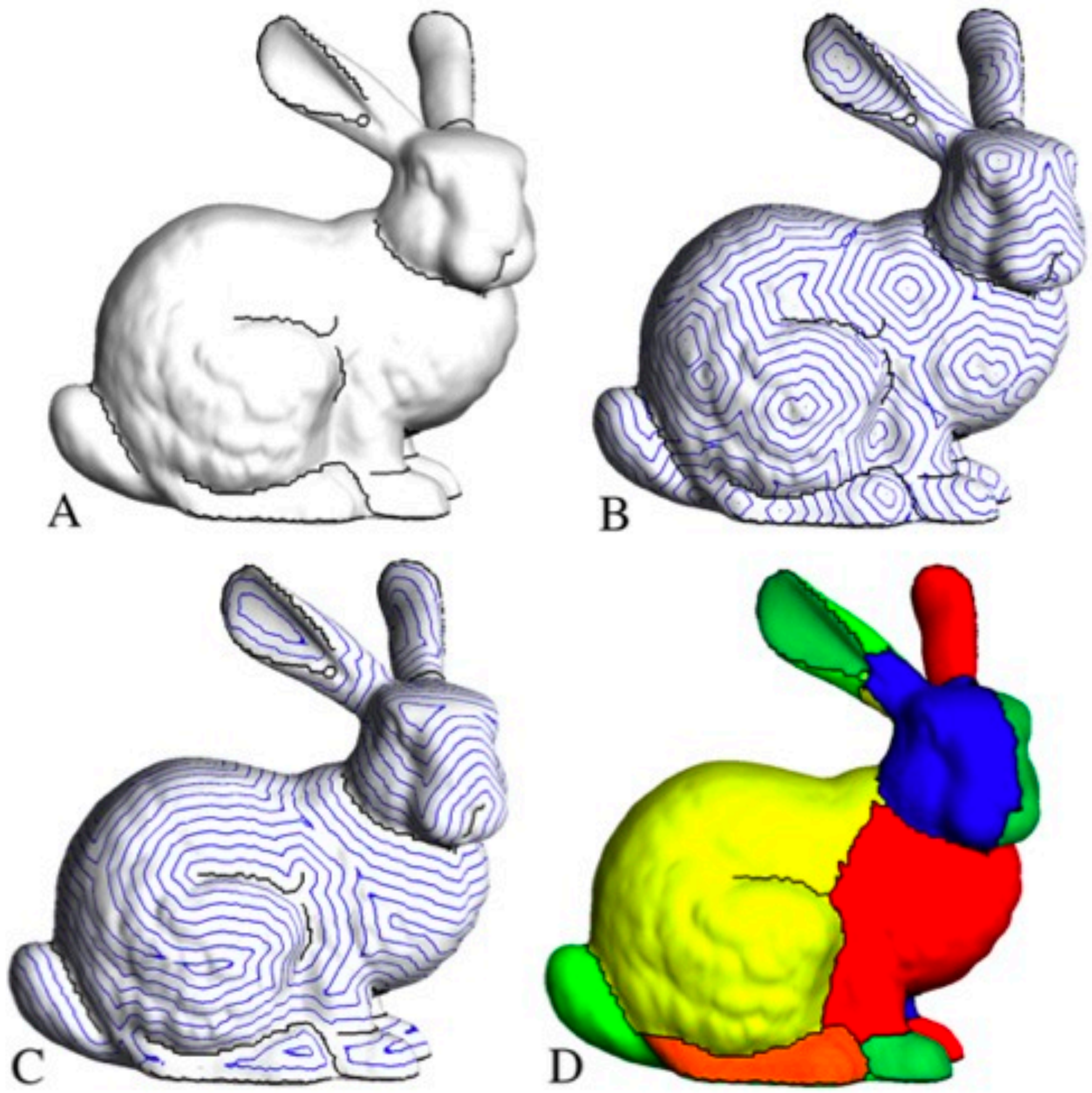
- We can ignore them and parametrize every chart separately
- We want to impose continuity of the derivatives of the parametrization across cuts
- Both have their own pros and cons, we will see an example for both cases

No continuity between charts

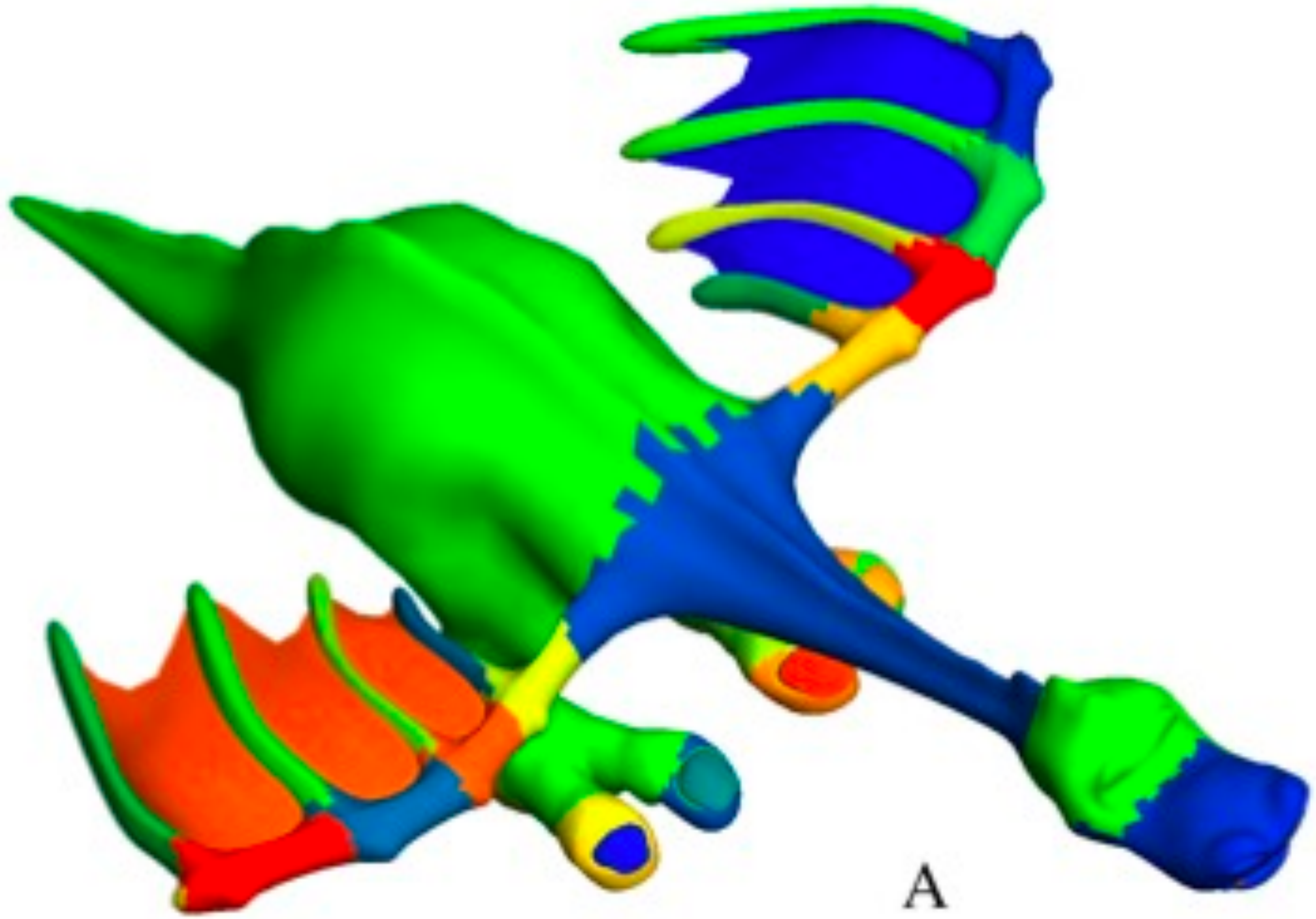
- Every chart can be parametrized separately
- The collection of the separate chart can be grouped in a square with a packing algorithm
- We study the method proposed in the LSCM paper

Segmentation

- Identify high curvature areas
- Seeds are the maxima of the distance to feature function
- Grow seeds into regions
- Merge regions if possible



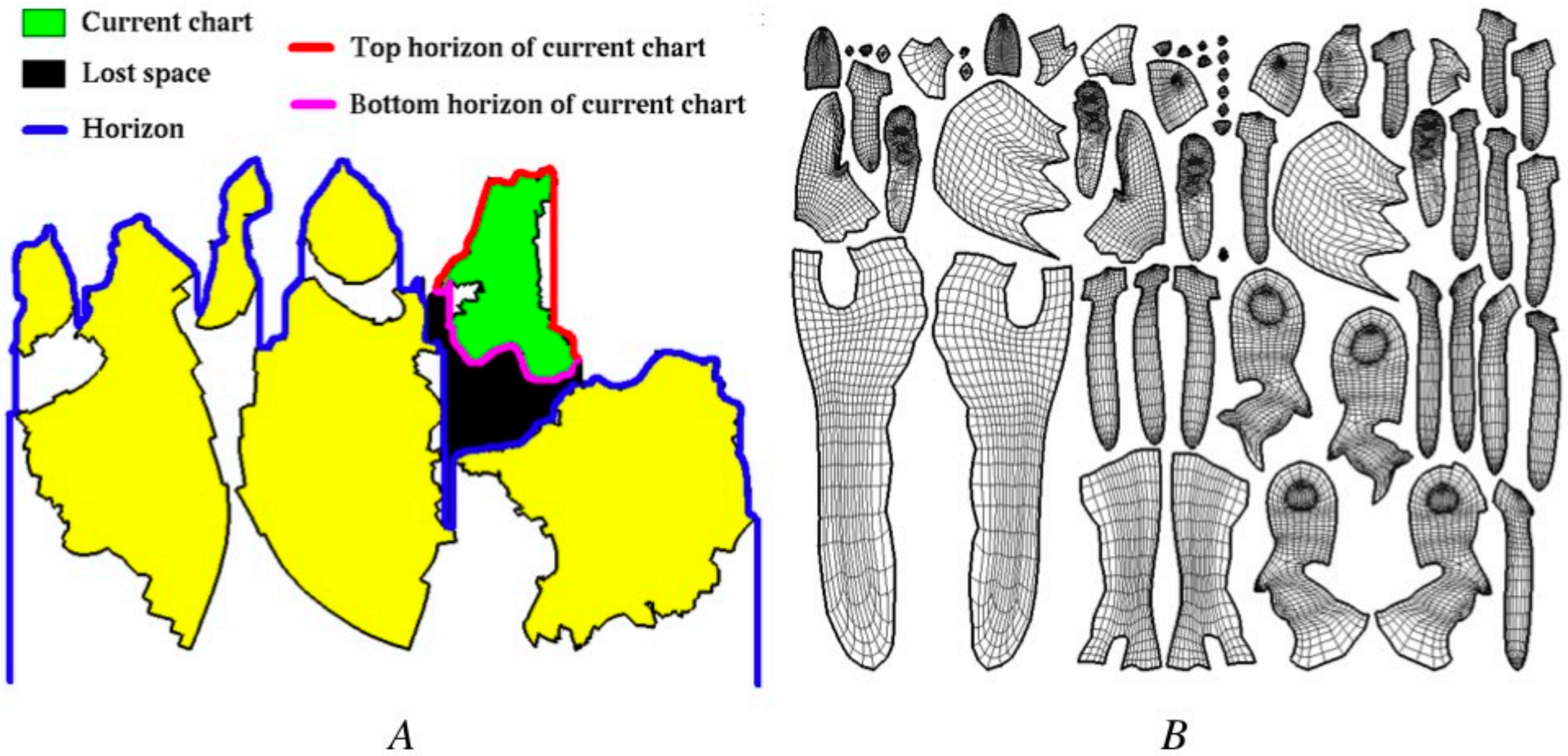
Segmentation result



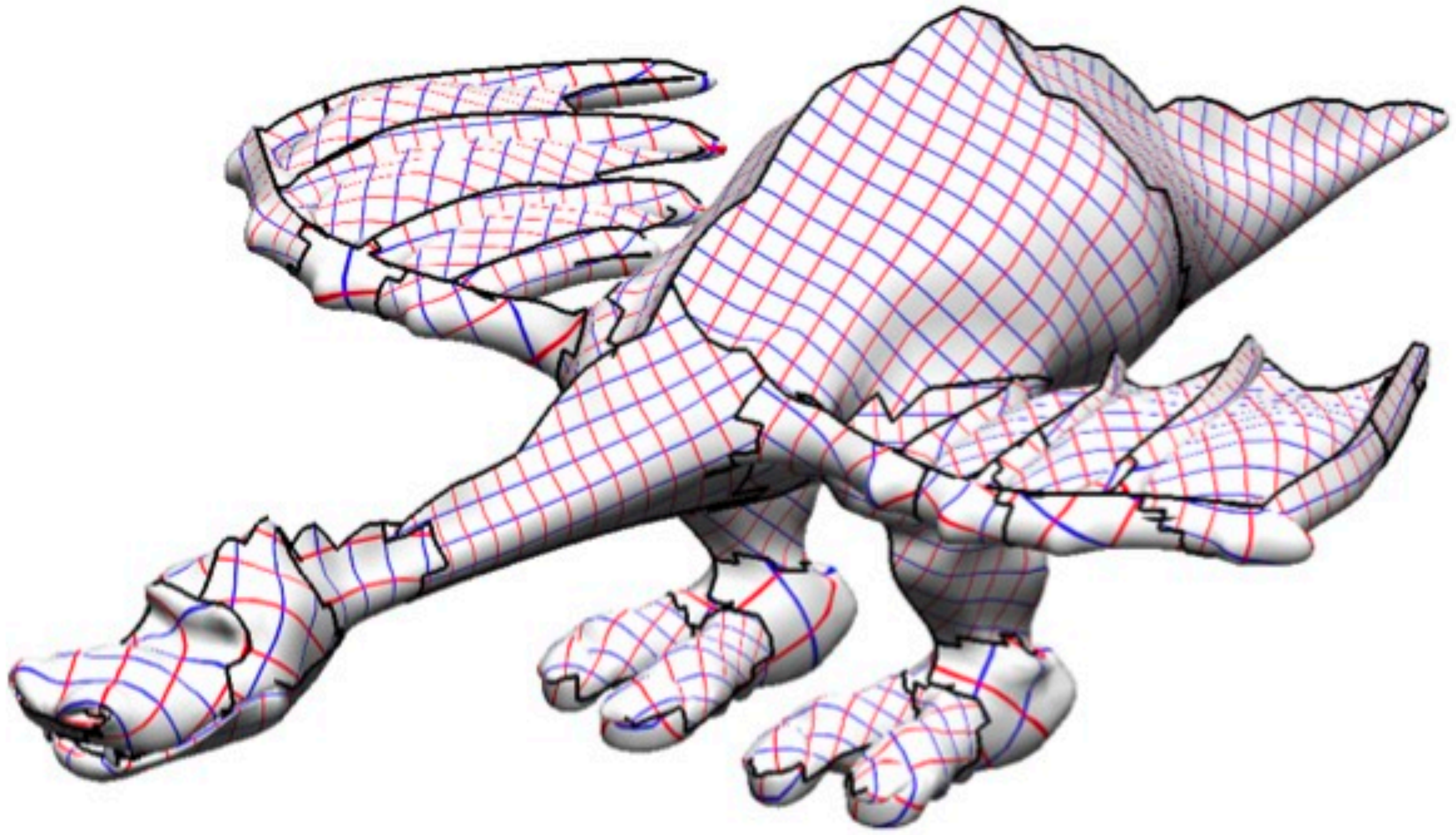
Packing

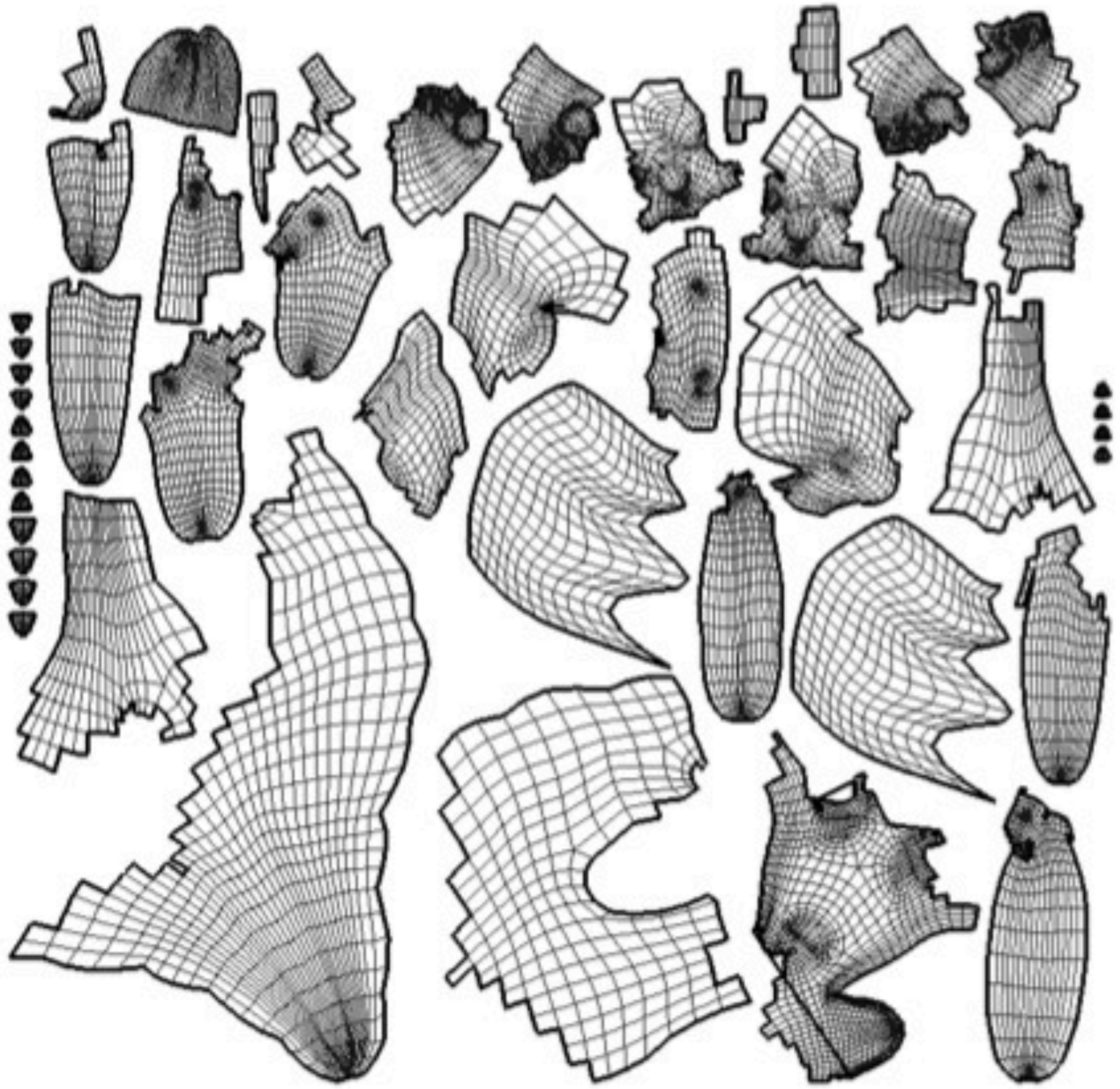
- The computation the optimal packing is NP-complete
- We need an heuristic
- The problem has been studied extensively in computational geometry, with the goal of computing high-quality results but only for a small number of objects
- The authors propose an heuristic based on the famous "Tetris" game that is extremely fast and generates good results

Tetris packing

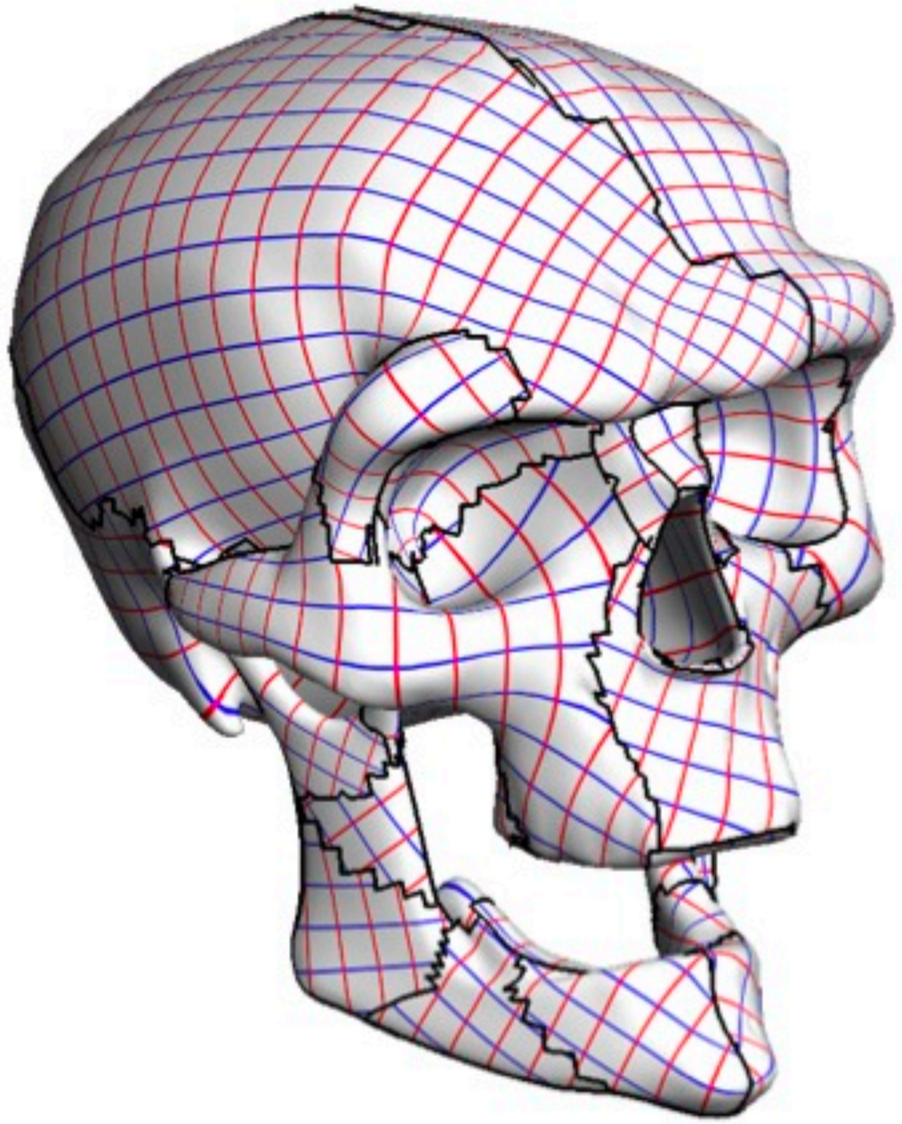


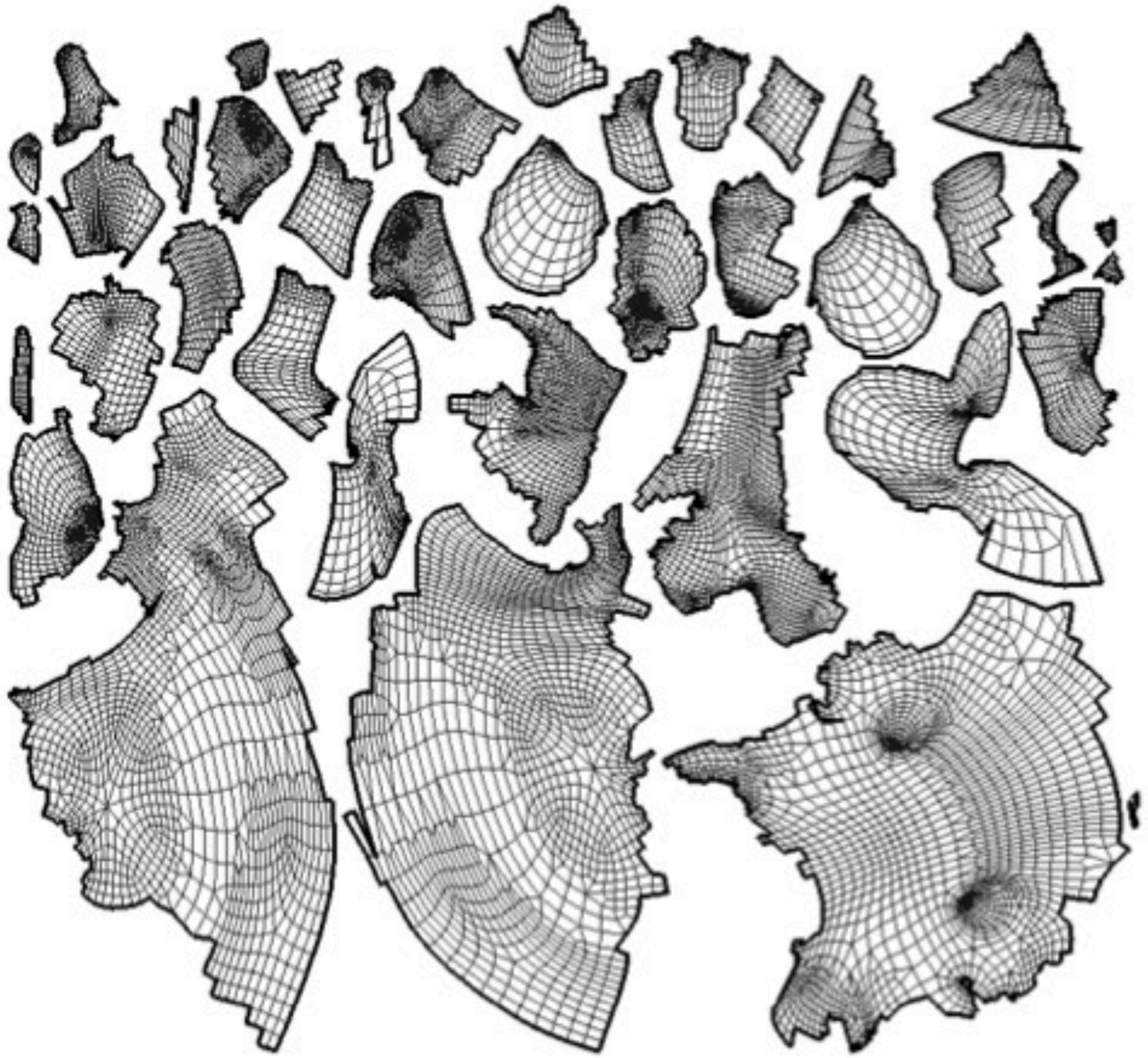
Results



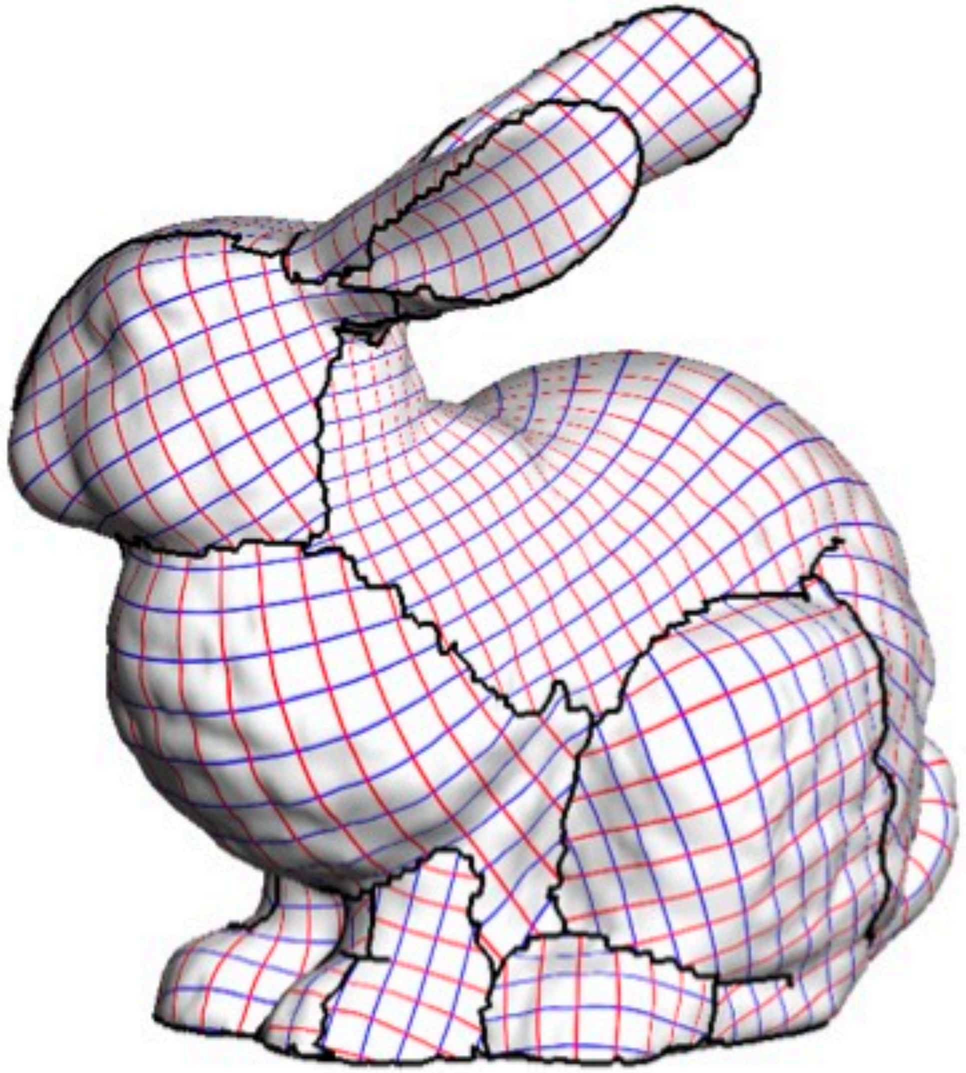


Results

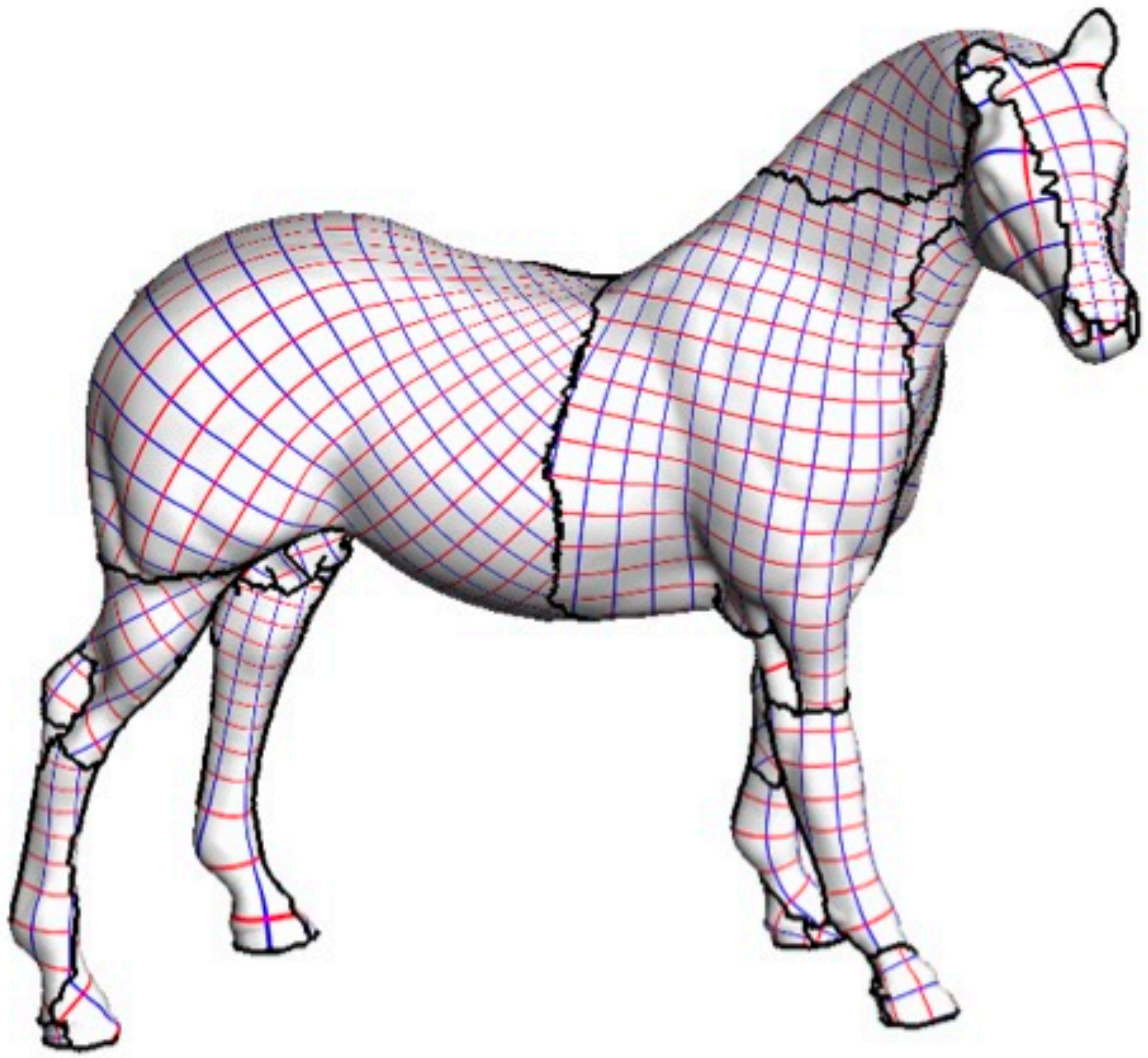




Results



Results



Applications

Continuity between charts

- For some applications (i.e. remeshing) continuity of the derivatives between charts is required
- We analyze a recent approach that is the current state of the art

Mixed-Integer Quadrangulation - David Bommes, Henrik Zimmer, Leif Kobbelt
Siggraph 2009

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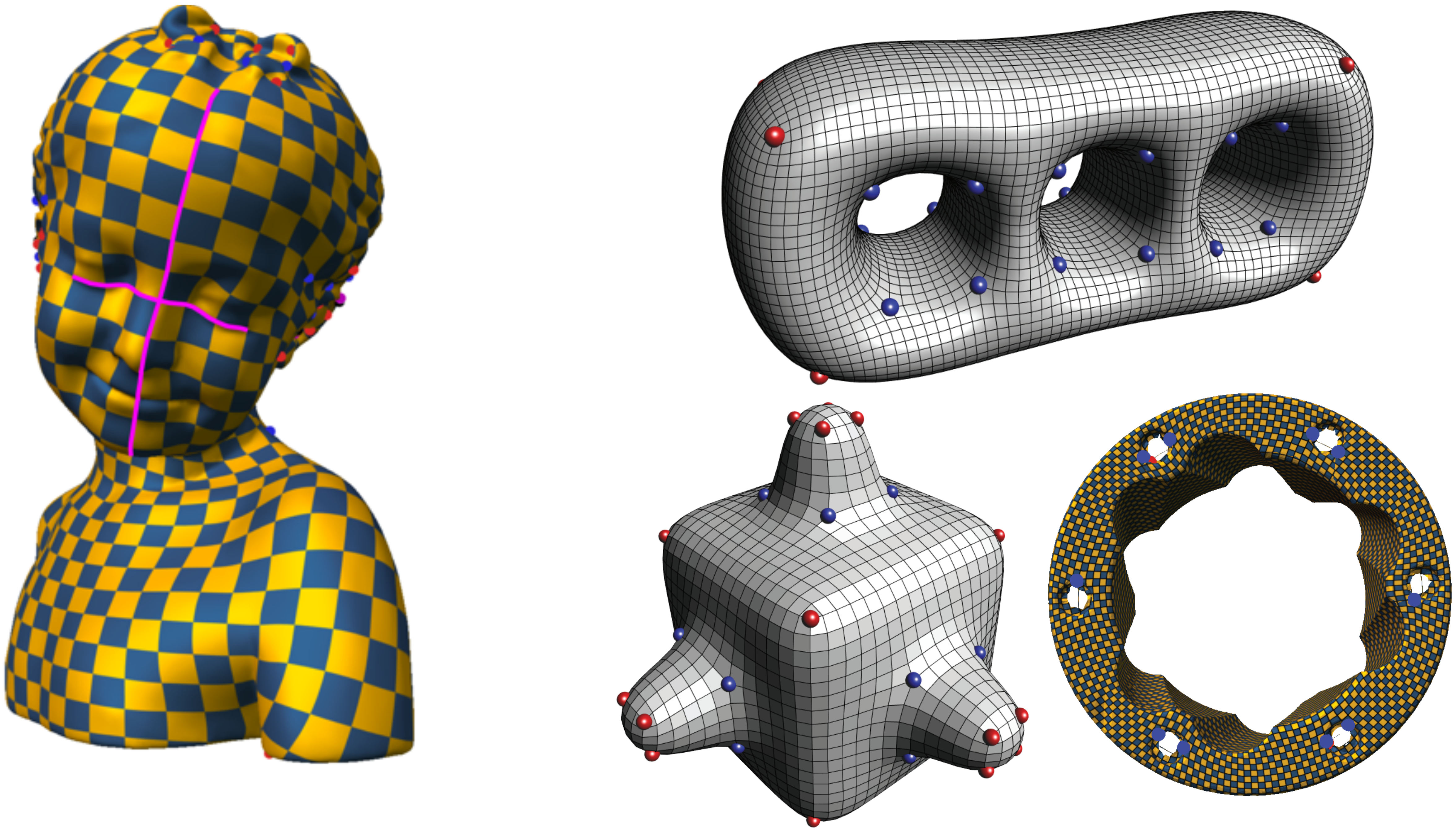
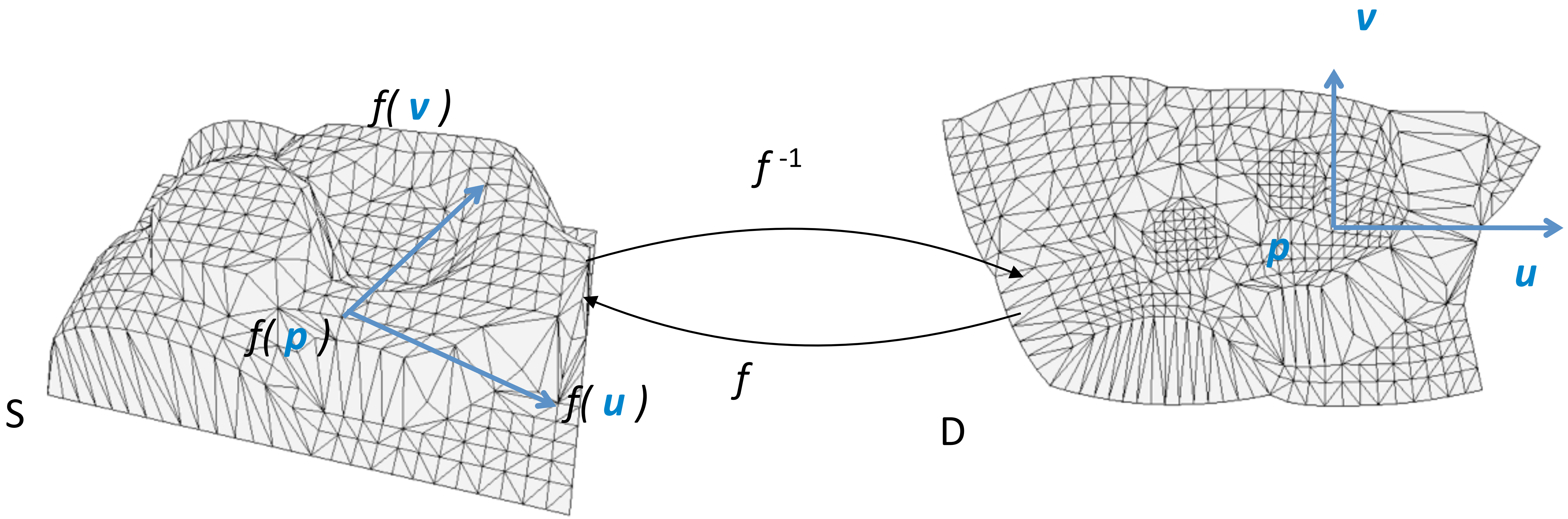
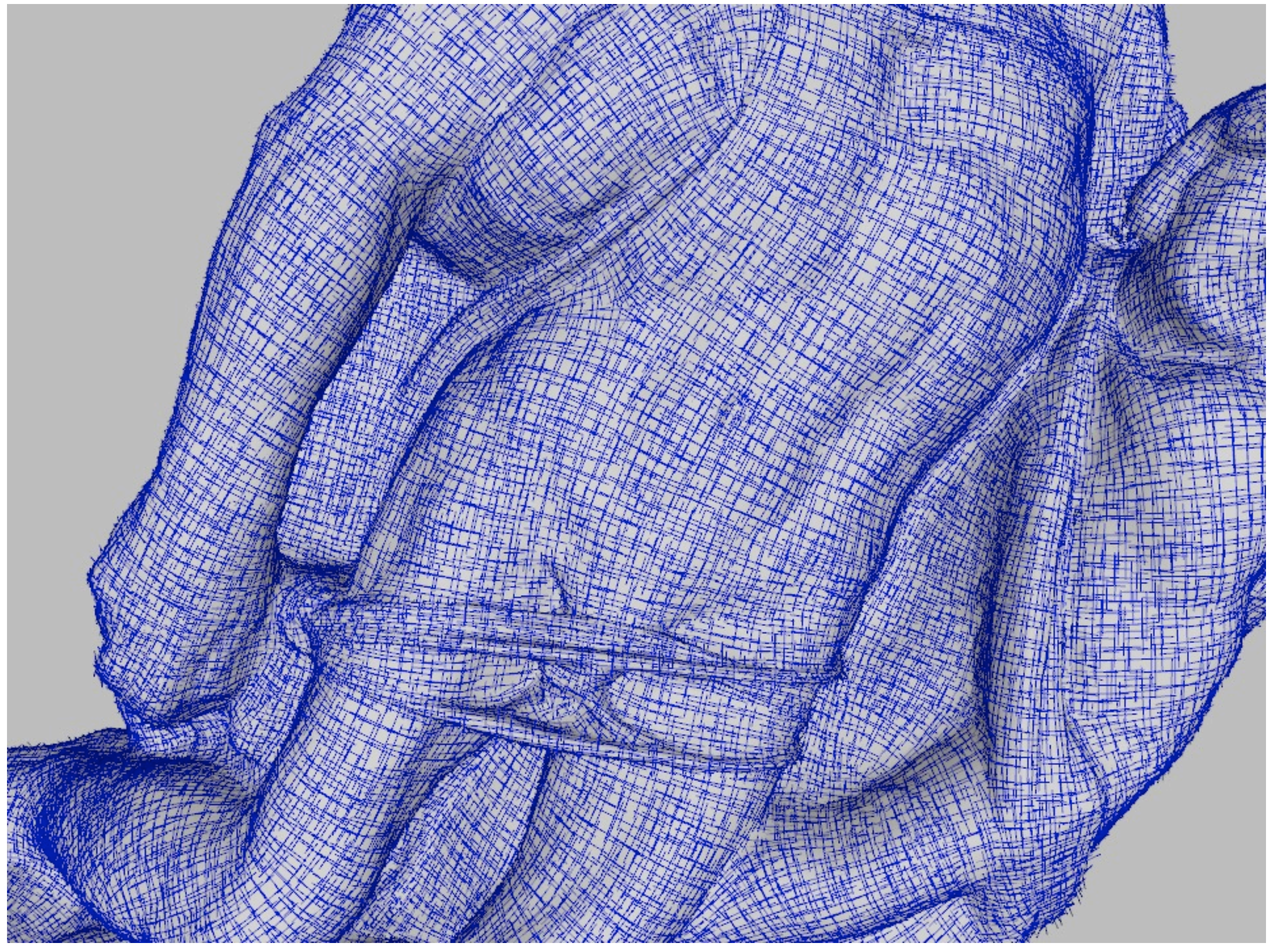
This is what we want to compute

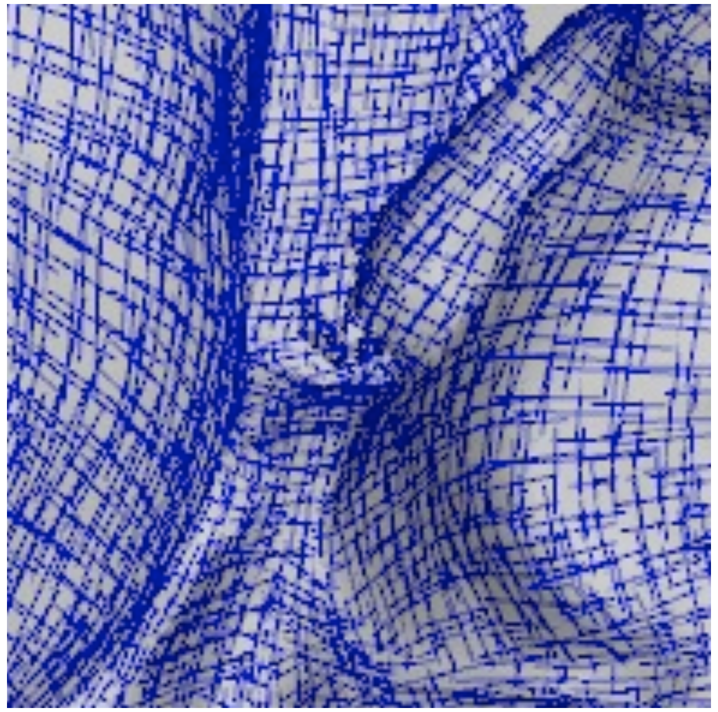


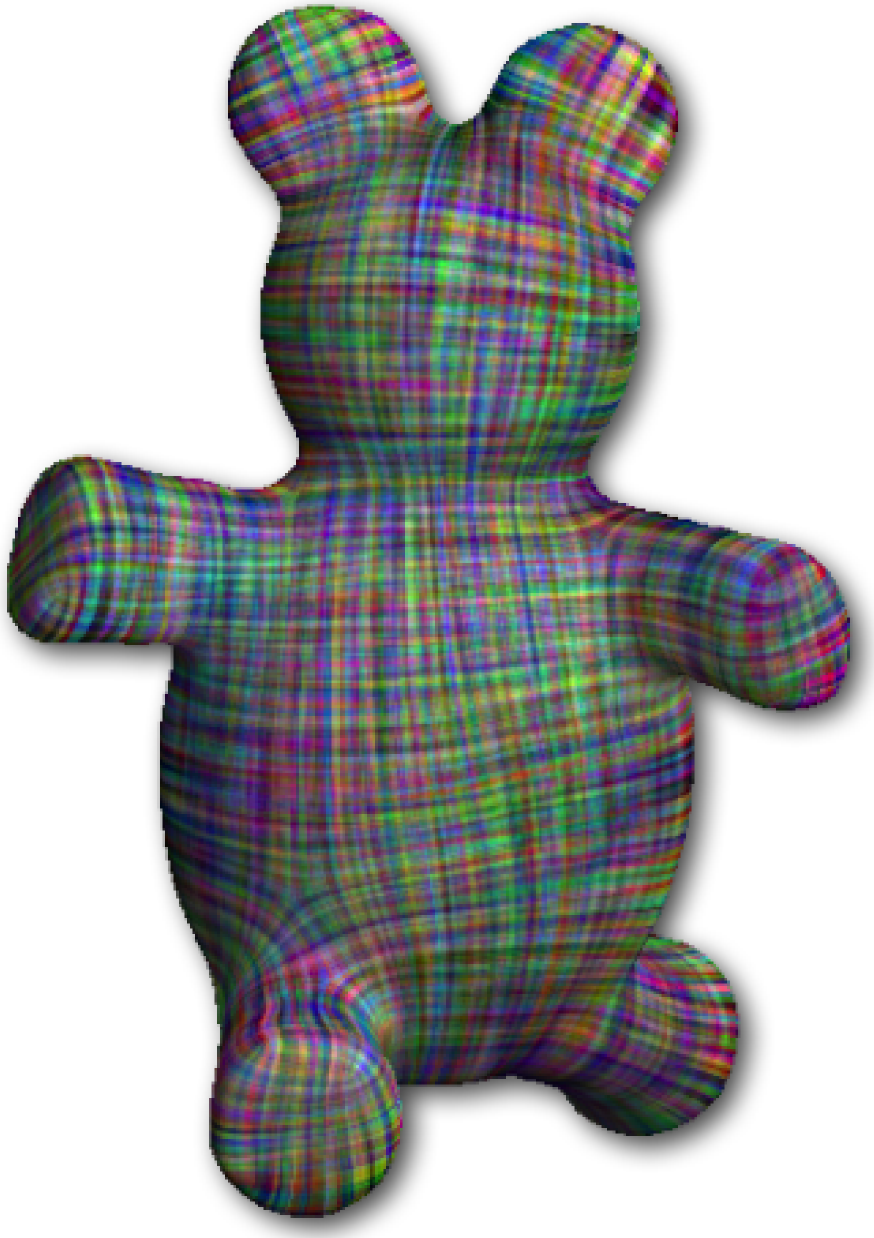
Inverse of a parametrization



Cross-fields







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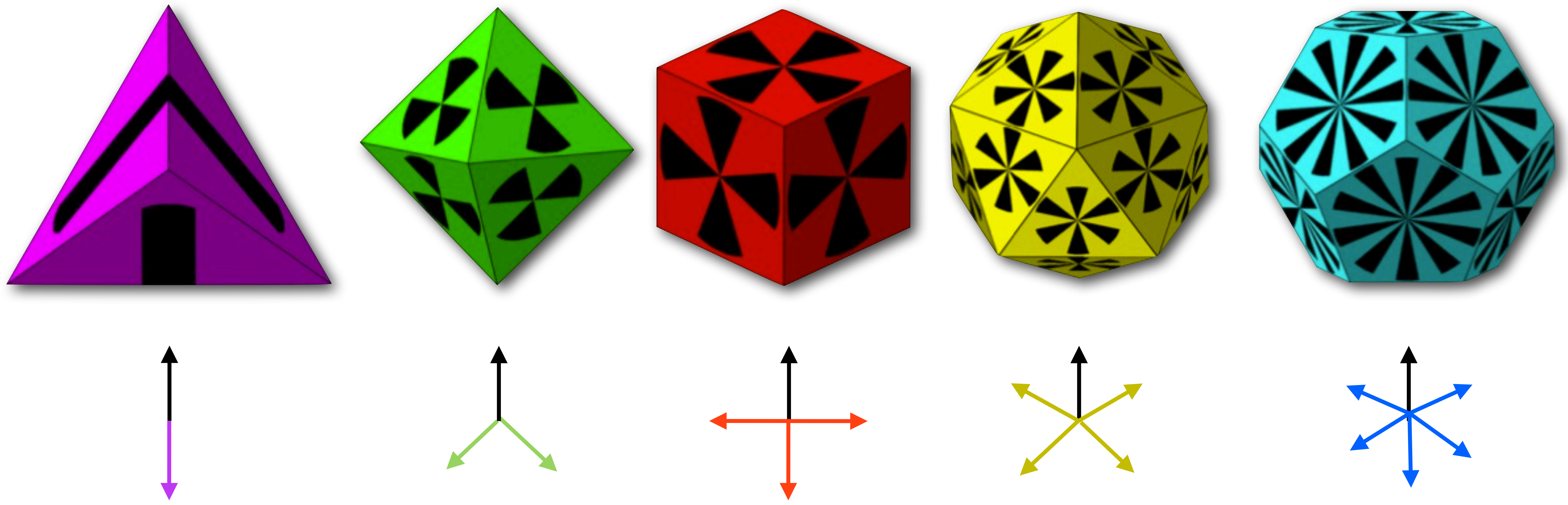
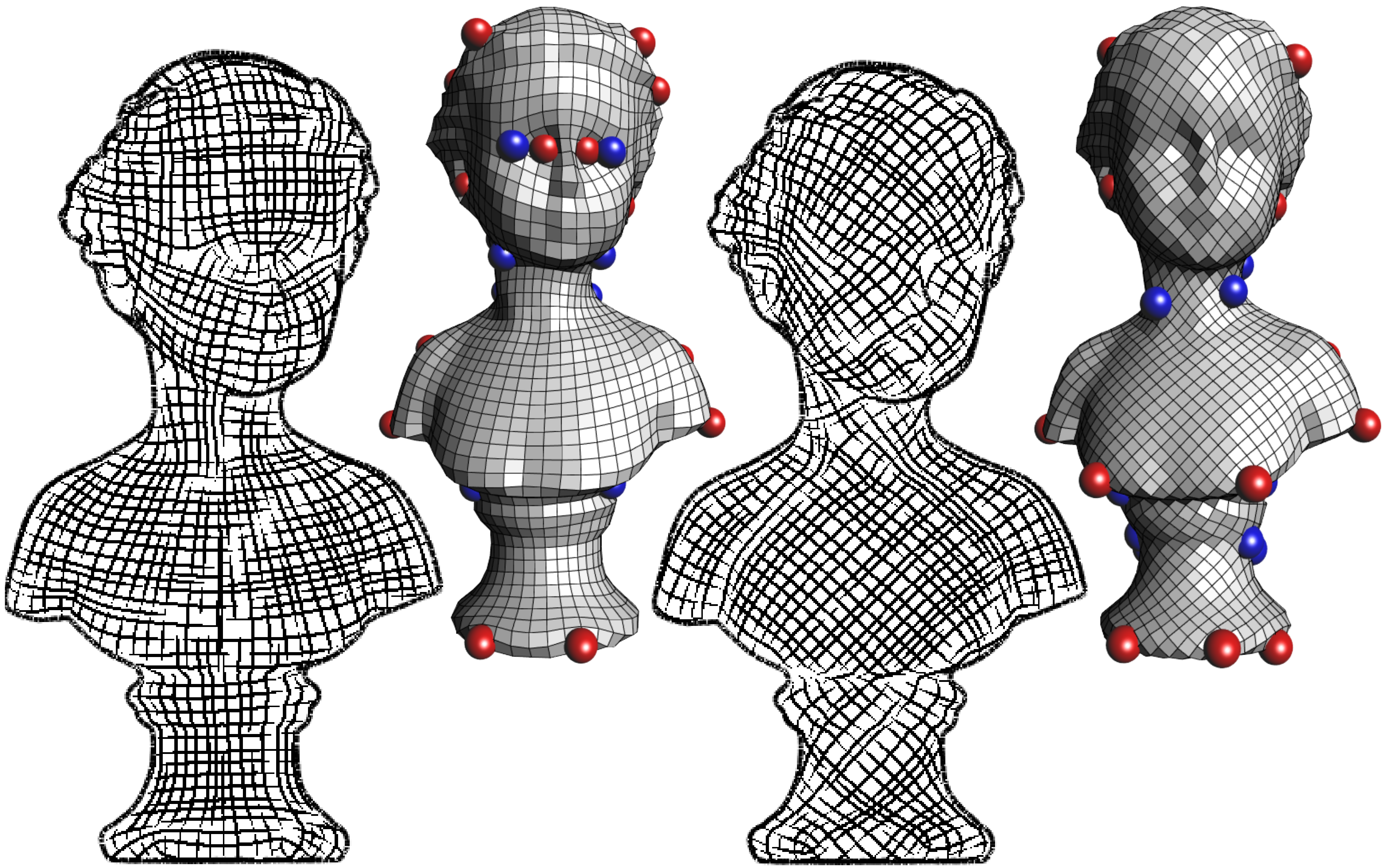
N-Rosy Fields



$\frac{2\pi}{2}$ $\frac{2\pi}{3}$ $\frac{2\pi}{4}$ $\frac{2\pi}{5}$ $\frac{2\pi}{6}$

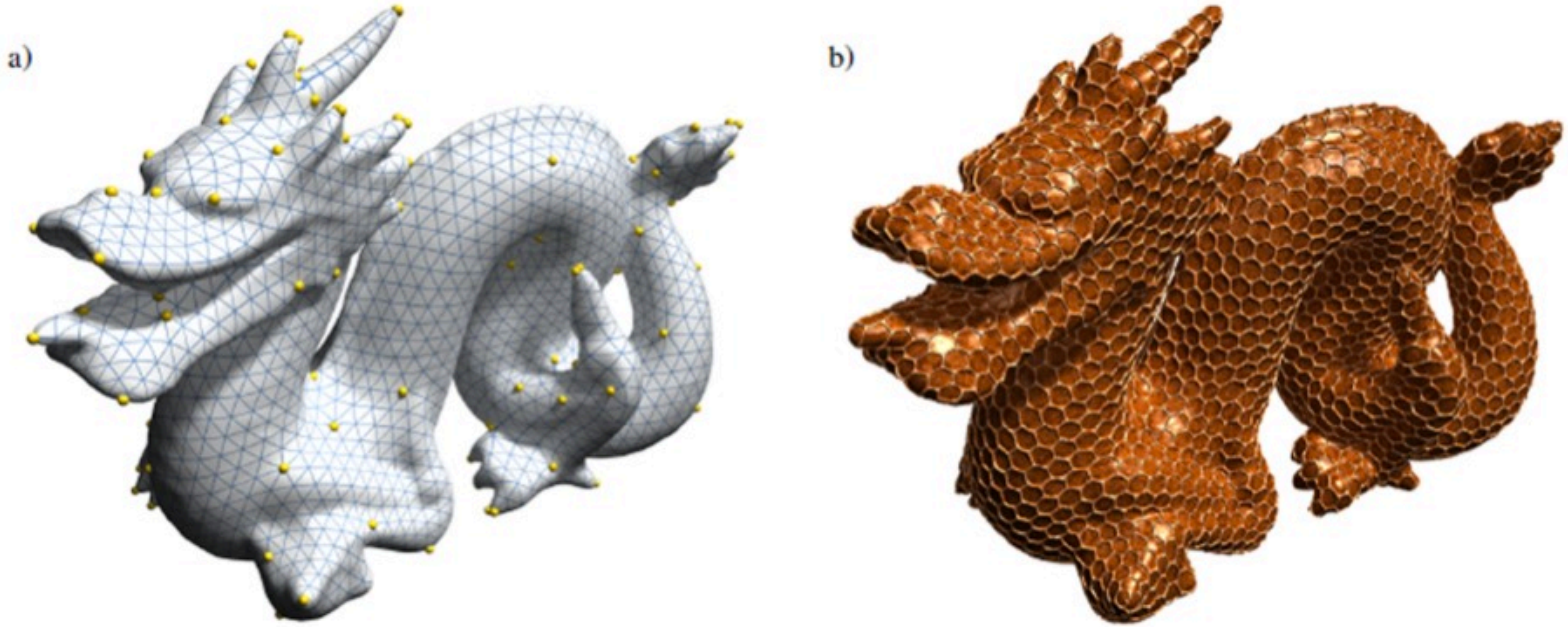
Rotational symmetry field design on surfaces - Jonathan Palacios, Eugene Zhang - Siggraph 2007

Applications of N-rosy



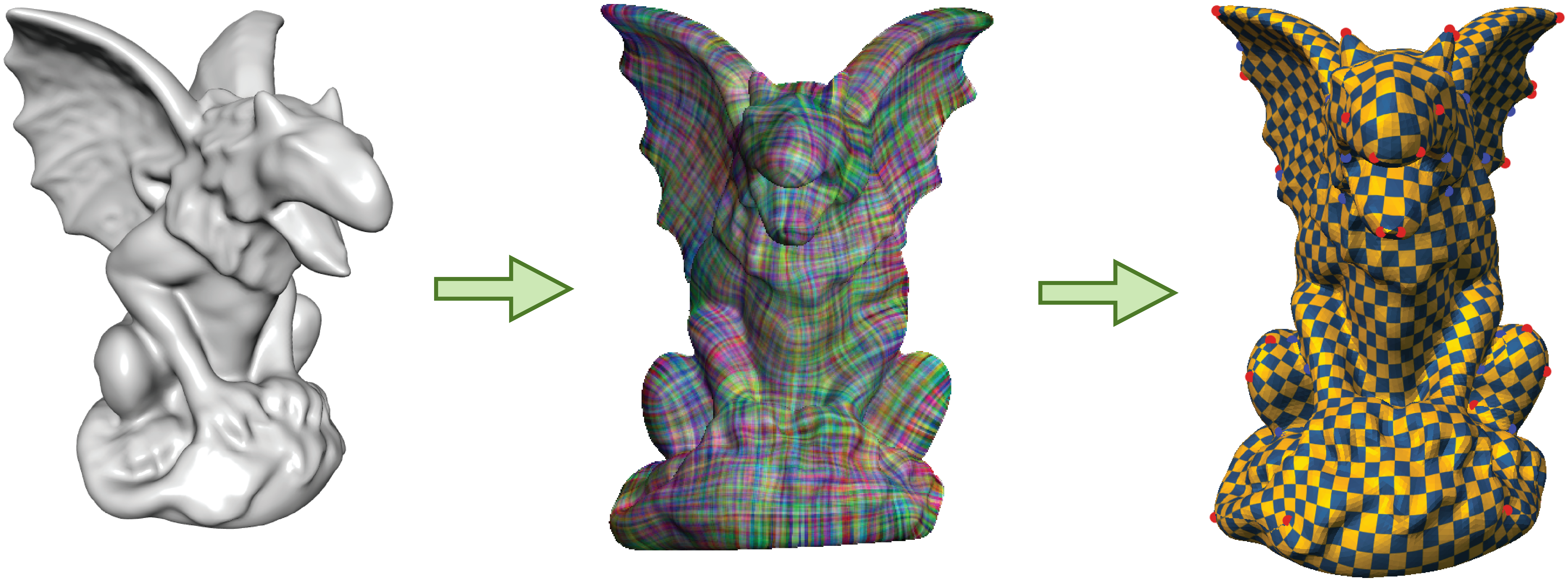
Illustrating smooth surfaces - Aaron Hertzmann, Denis Zorin - Siggraph 2000

Hexagonal remeshing

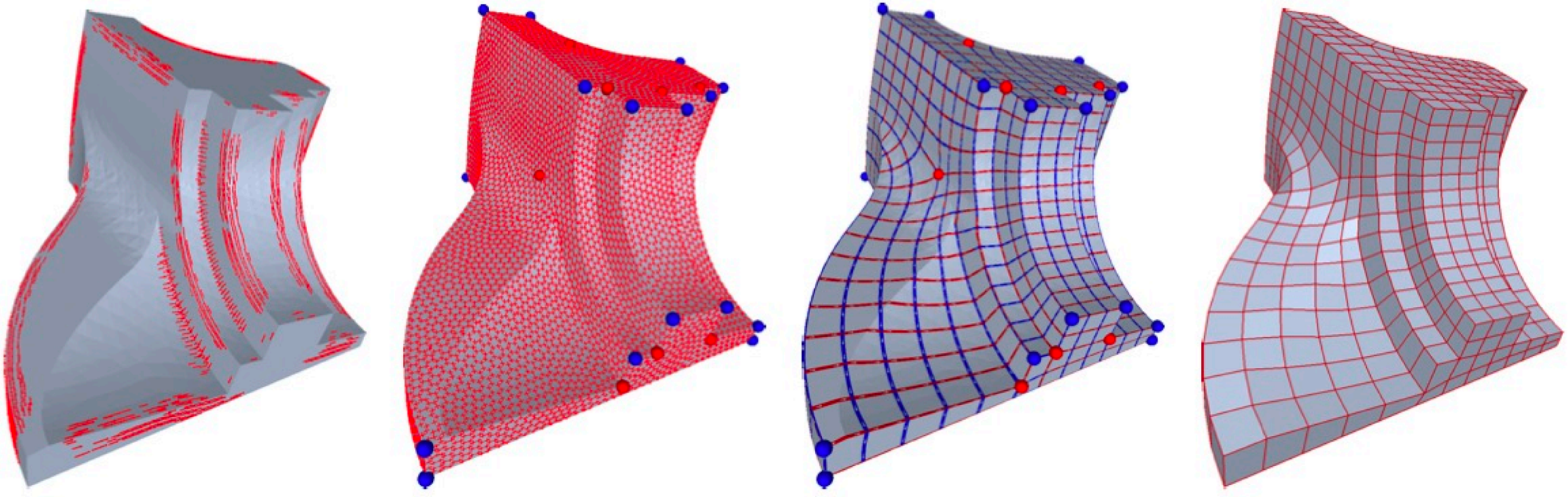


Hexagonal Global Parameterization of Arbitrary Surfaces [Nieser et all 2010]

Cross-fields based parametrization



Mixed-integer quadrangulation

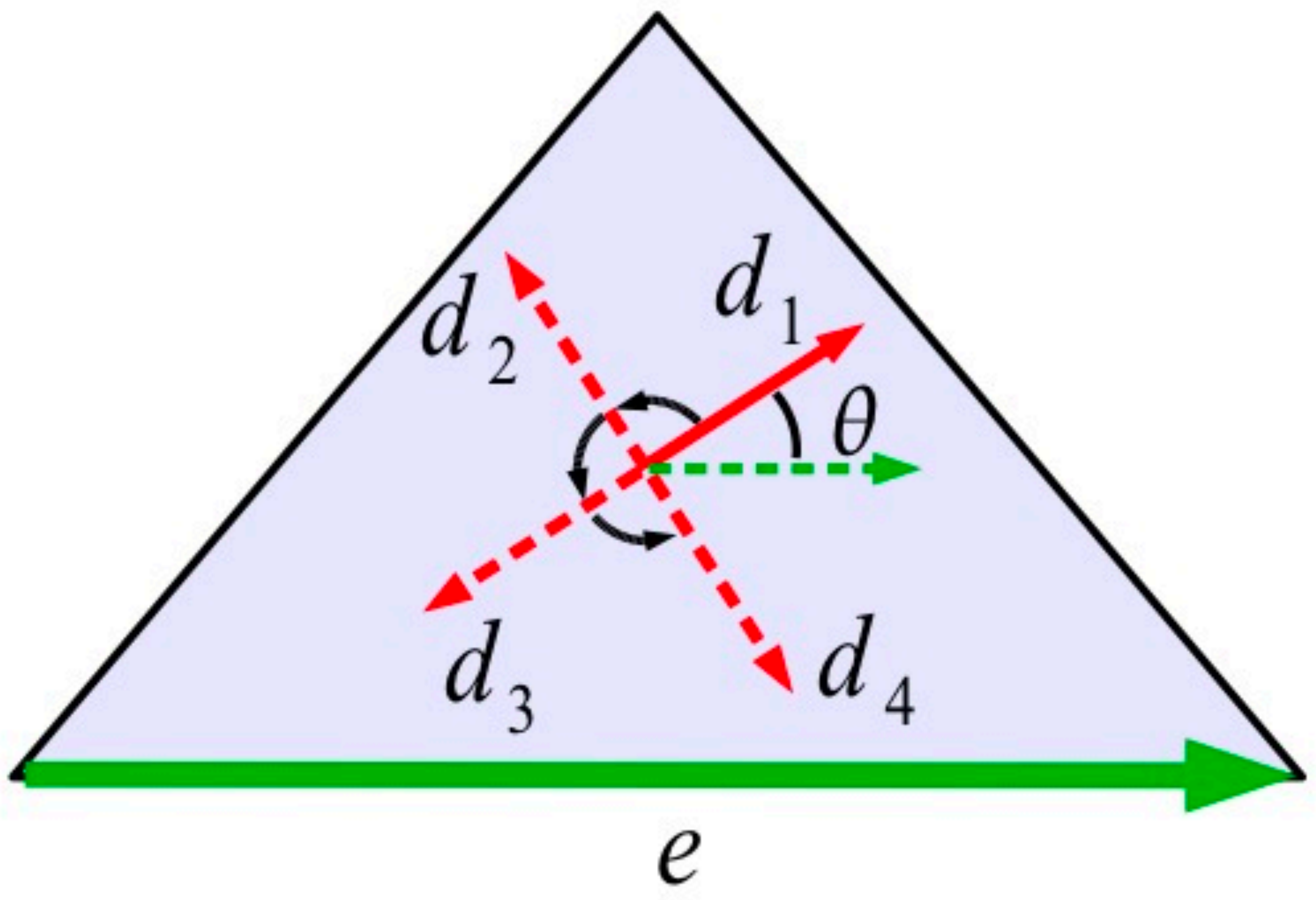


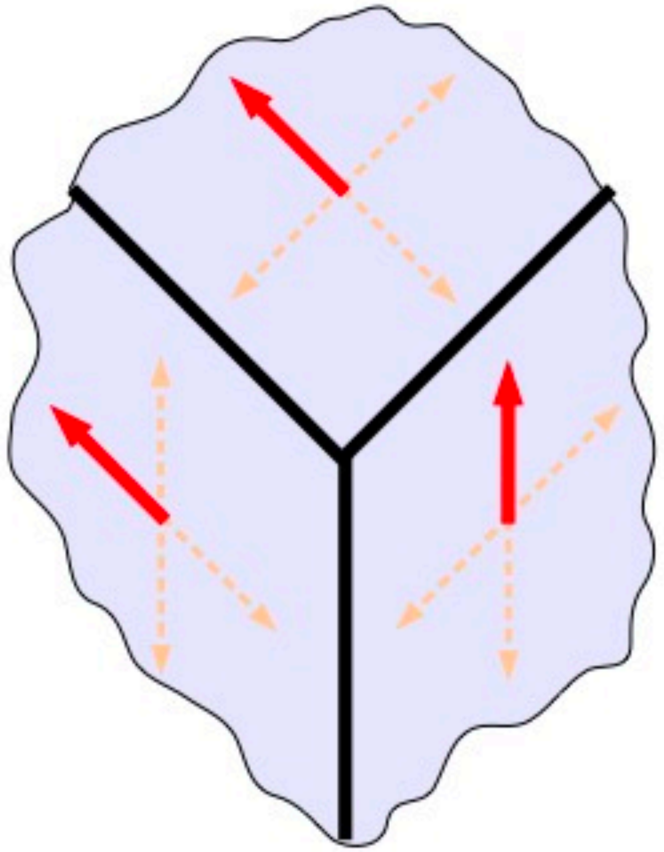
Feature line extraction

- Curvature analysis on the surface and hard thresholding
- The set of feature can be sparse or dense, the algorithm will automatically adapt and produce a result that satisfy the constraints
- The quality of the results depends completely and non-smoothly on the choice of the features

Field representation

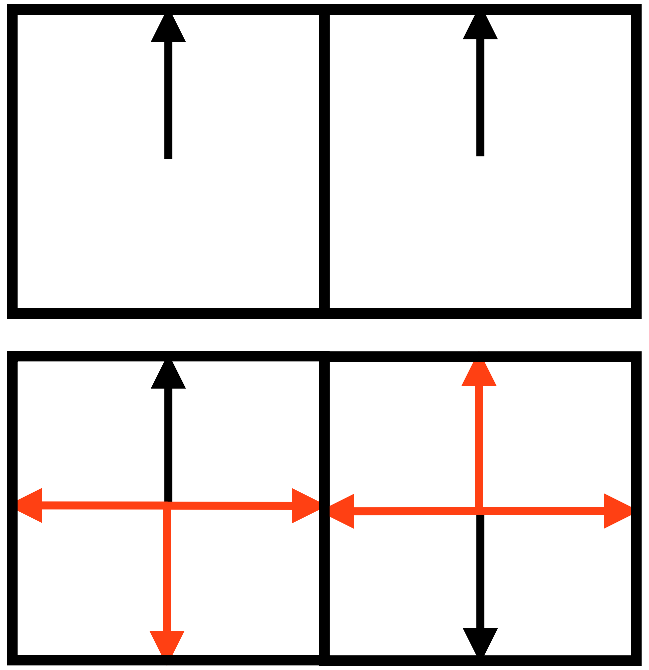
- The field is discretized on faces
- The field representation is NOT unique





Field smoothing

- MI minimizes a smoothness energy with integer variables in it



Field smoothness energy on a planar triangulation

$$E_{smooth} = \sum_{e_{ij} \in E} (\theta_i - \theta_j)^2$$

Cross-Field smoothness energy on a mesh

$$E_{smooth} = \sum_{e_{ij} \in E} (\underbrace{\theta_i + \kappa_{ij} + \frac{\pi}{2} p_{ij}}_{\theta_i \text{ w.r.t. frame } j} - \theta_j)^2$$

Integer variables?

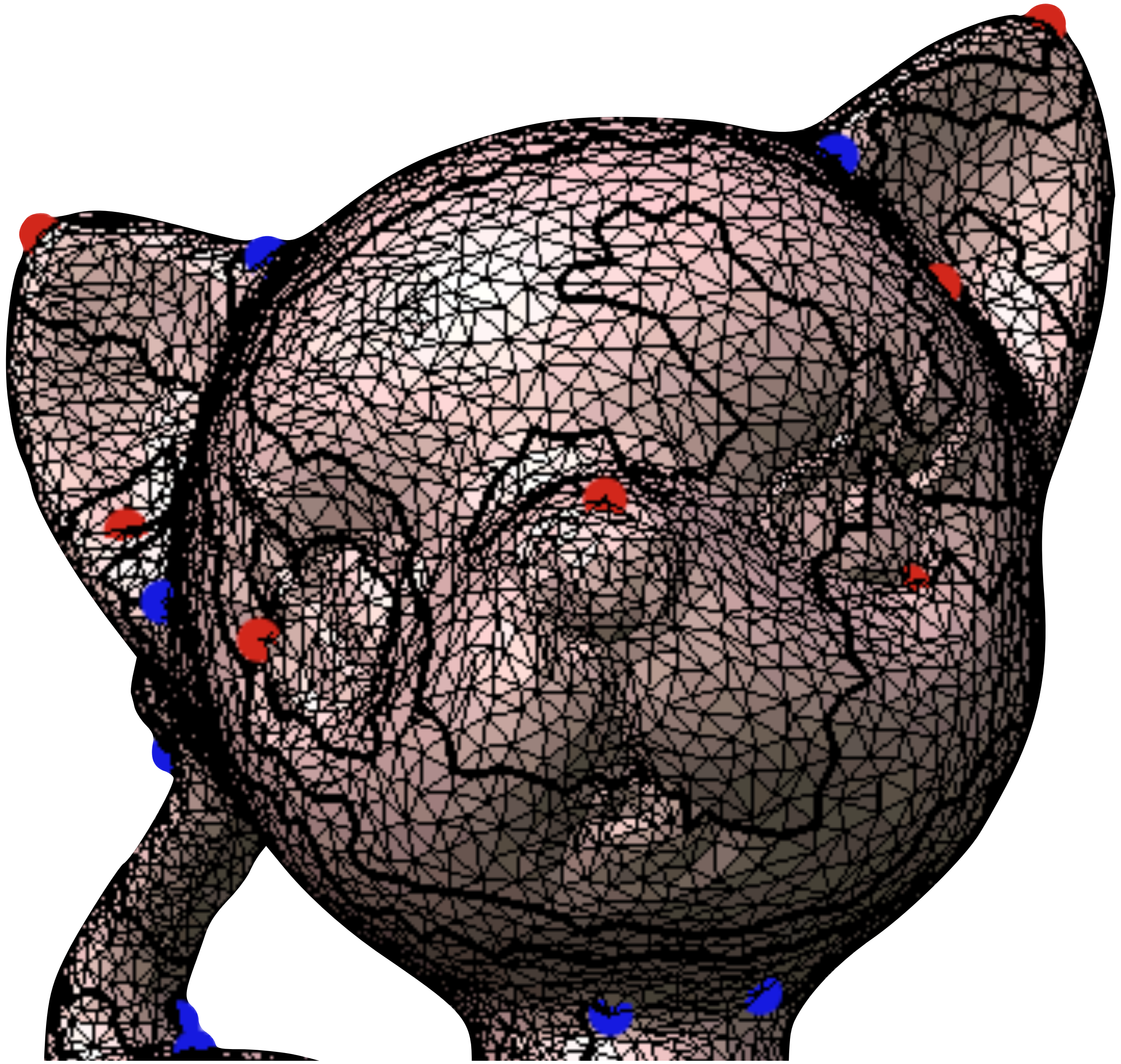
- Numerical minimization problems with real and integer variables are common and there are no fast ways of solving them in general
- If the integer variables can be transformed into real variables and they still make sense, then it is possible to solve the problem iteratively:
  - Consider all variables real and minimize
  - Look in the solution for the number that is as close as possible to an integer and that correspond to an integer variable. Round it to the closest integer and consider it constant
  - Minimize the energy again and iterate until all the integer variables are fixed

Mixed-integer parametrization

- Once the cross-field is computed, it is used to define a parametrization
- The full parametrization is computed with a single mixed-integer minimization

Cuts

- The cuts are automatically computed
- It starts from a random triangle and build a spanning tree that is then pruned to compute the cut graph



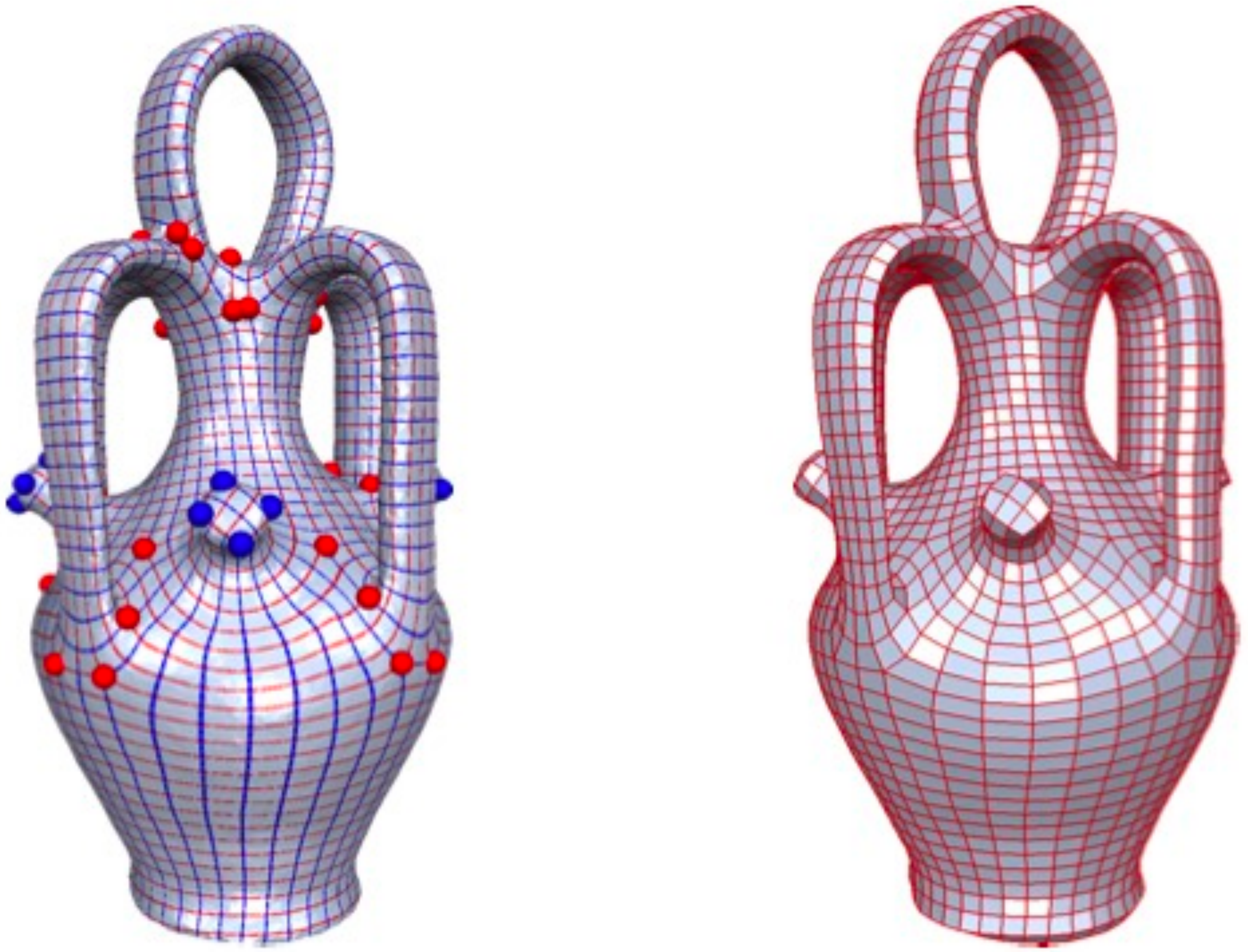
Parametrization optimization

- The parametrization gradient should be aligned with the cross-field

$$E_T = ||h\nabla u - \mathbf{u}_T||^2 + ||h\nabla v - \mathbf{v}_T||^2$$

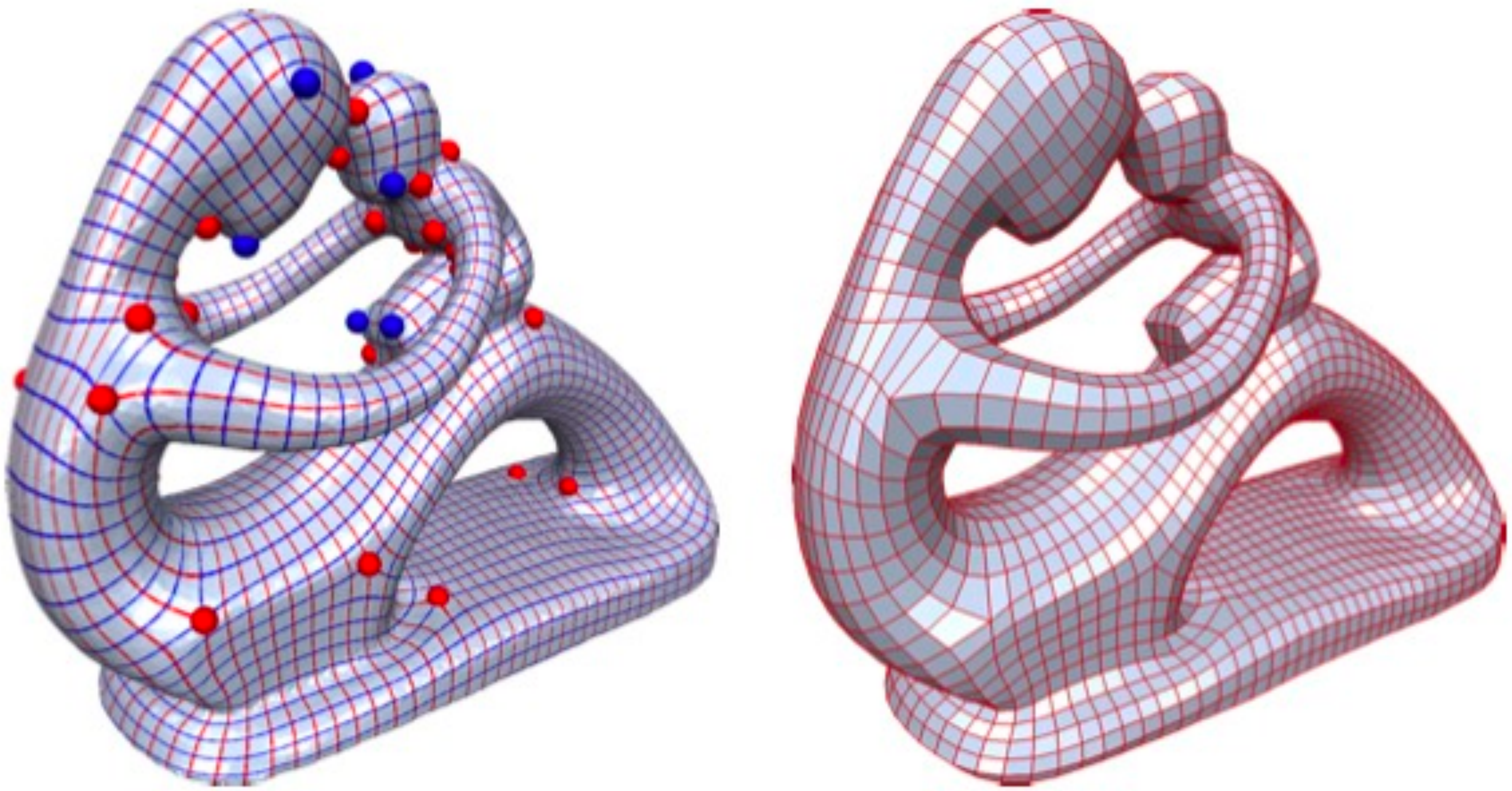
- Constraints are added to force every cutted edge to be orizontal or vertical
- Constraints are added to force every cutted edge to match the direction of his copy up to a 90 degrees rotation and an integer translation

Results



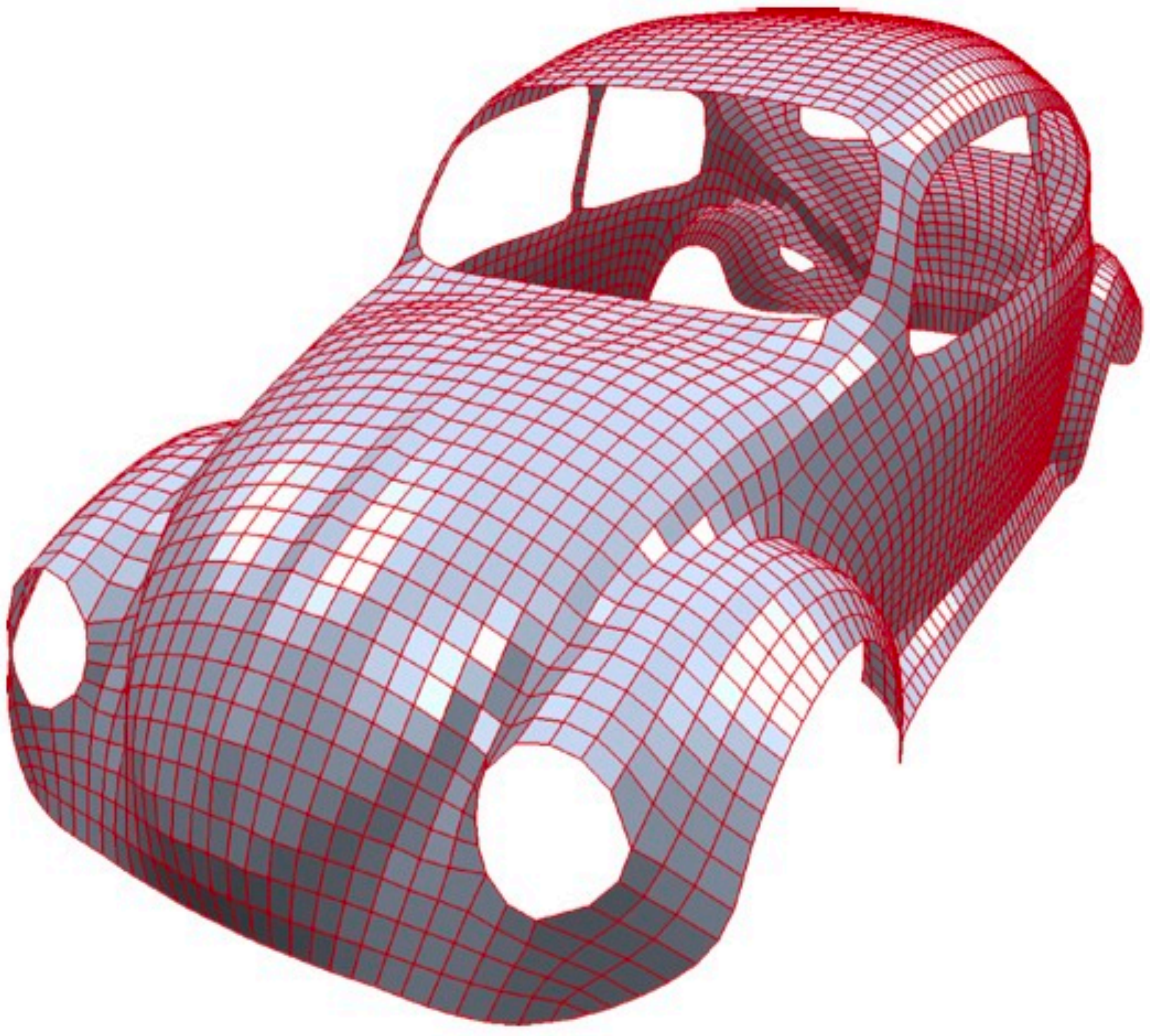
Botijo

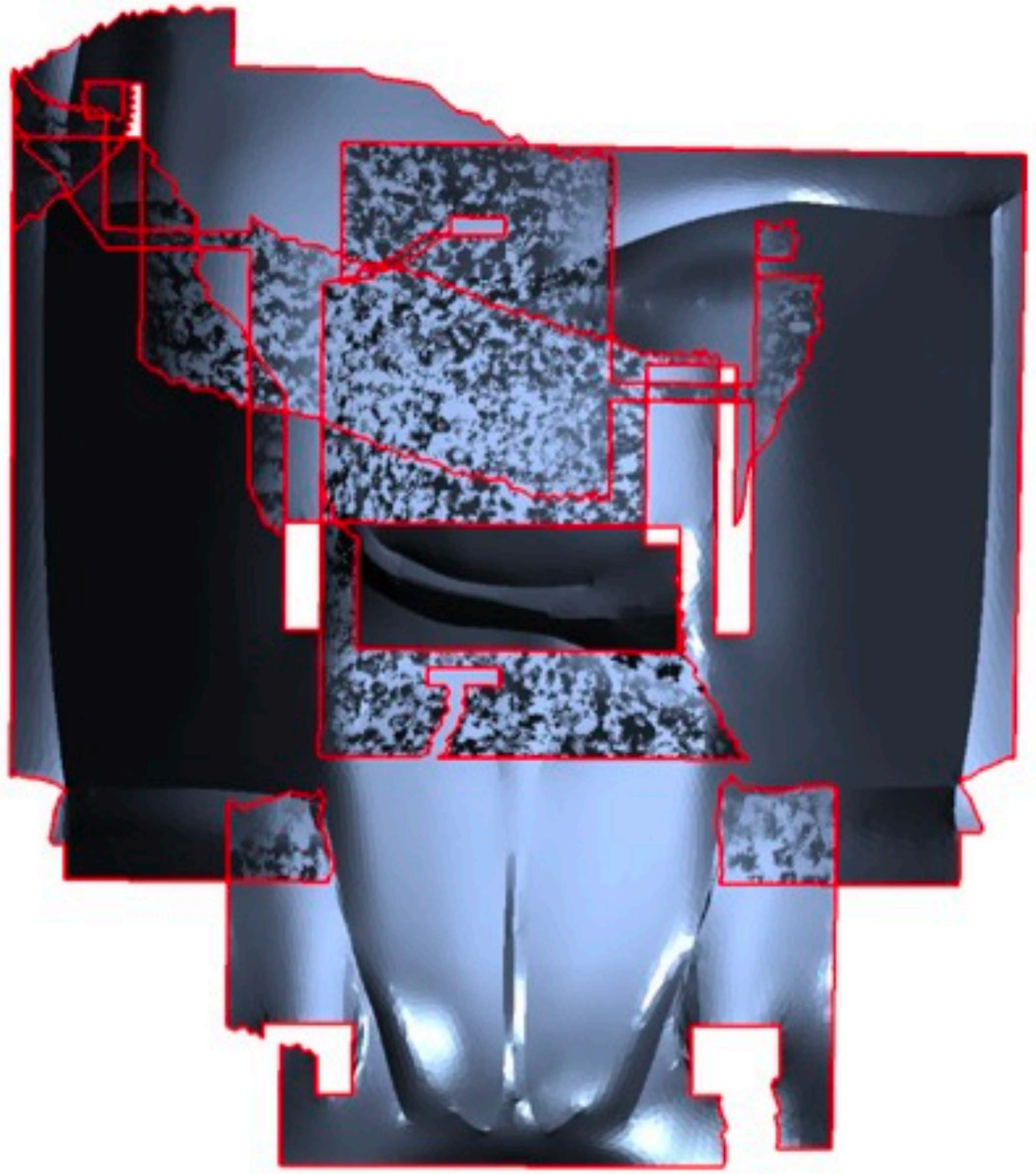
Results



FERTILITY

Results





Conclusions

- Mesh parametrization is still an open problem with an active community working on it
- For single patch mesh parametrization the main challenge is the computation of low-distortion parametrization with free-boundary and no overlaps
- There are currently no completely automatic methods that always works and produce good results. In the industry usually these methods are used as a starting point and then refined manually. For example, 3D Coat is using a variant of Mixed-Integer to generate high-quality quadrangulation

http://www.youtube.com/watch?v=amdKSfEmd0U&list=PLF62F3C31216D2794&index=1&feature=plpp_video