

CS 6501

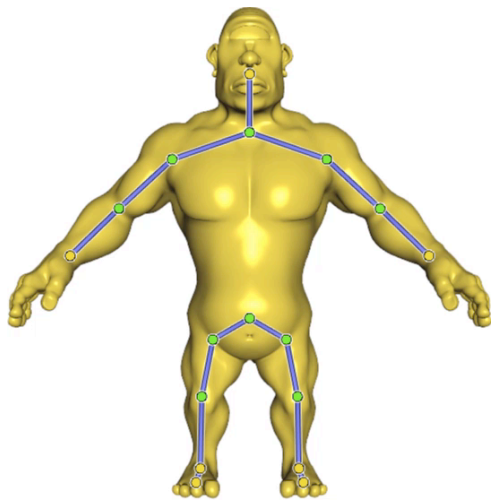
# 2D/3D Shape Manipulation, 3D Printing

---

## Linear Blend Skinning

Slides from Olga Sorkine

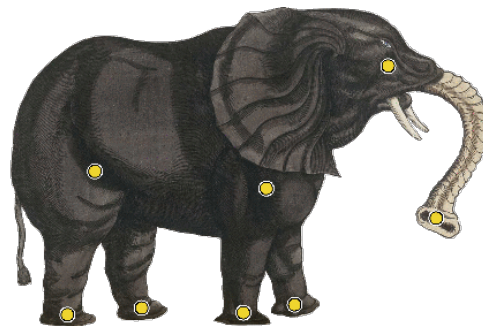
# LBS generalizes to different handle types



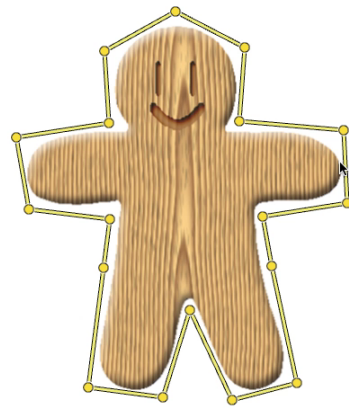
skeletons



regions

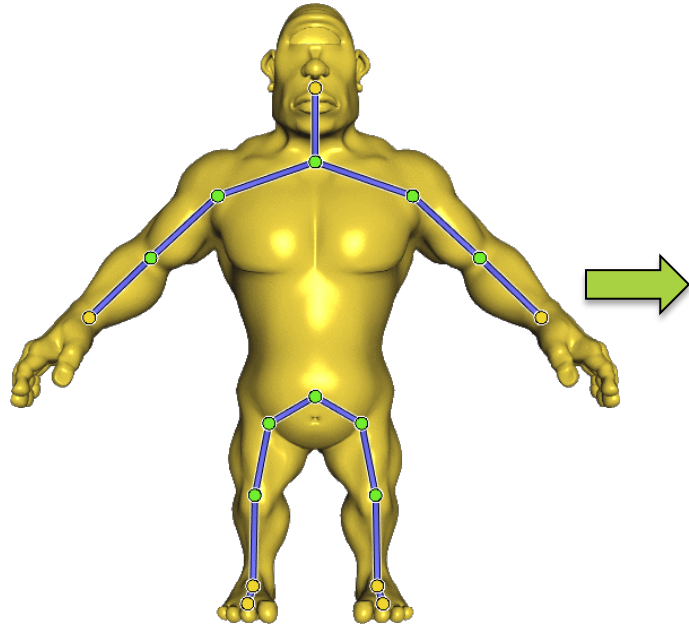


points



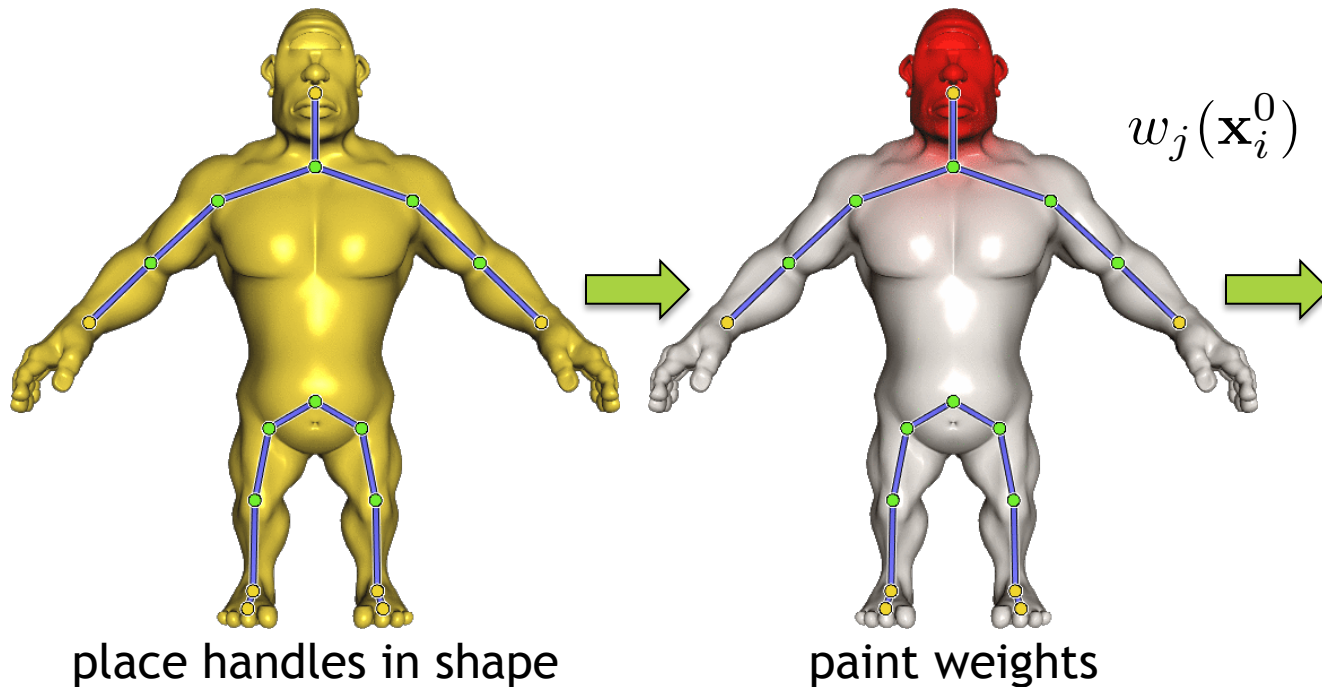
cages

# Linear Blend Skinning rigging preferred for its real-time performance



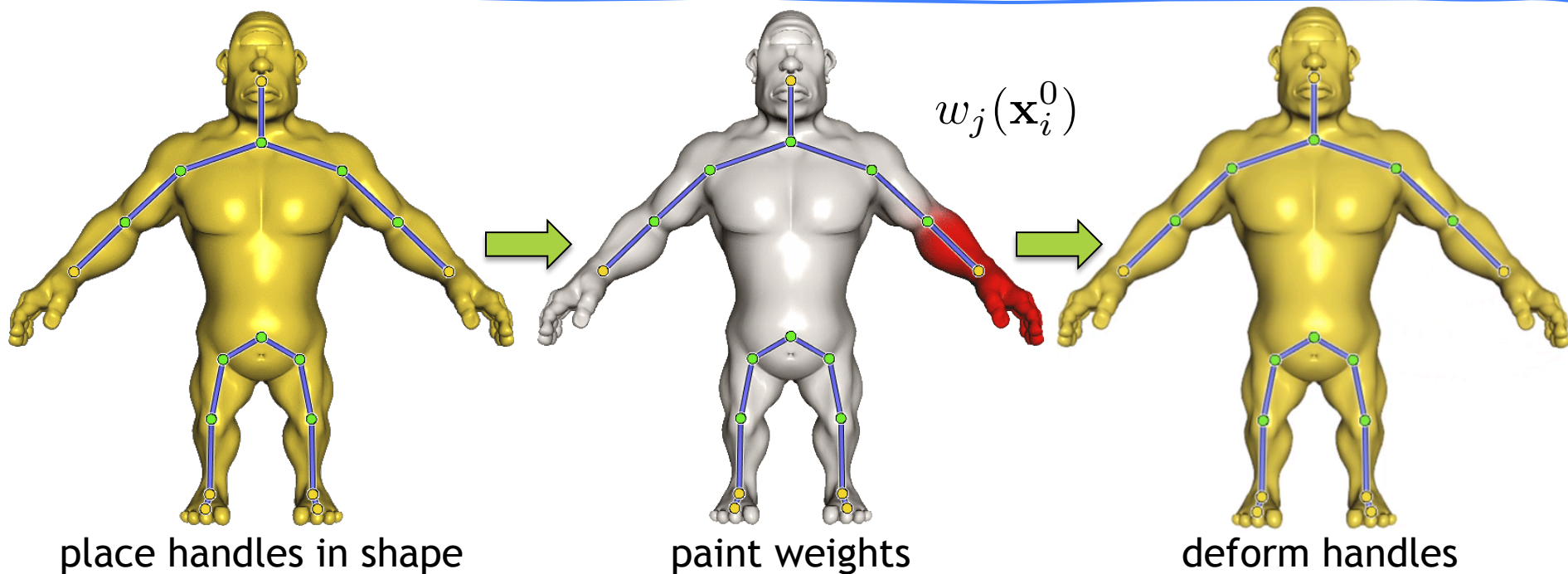
place handles in shape

# Linear Blend Skinning rigging preferred for its real-time performance

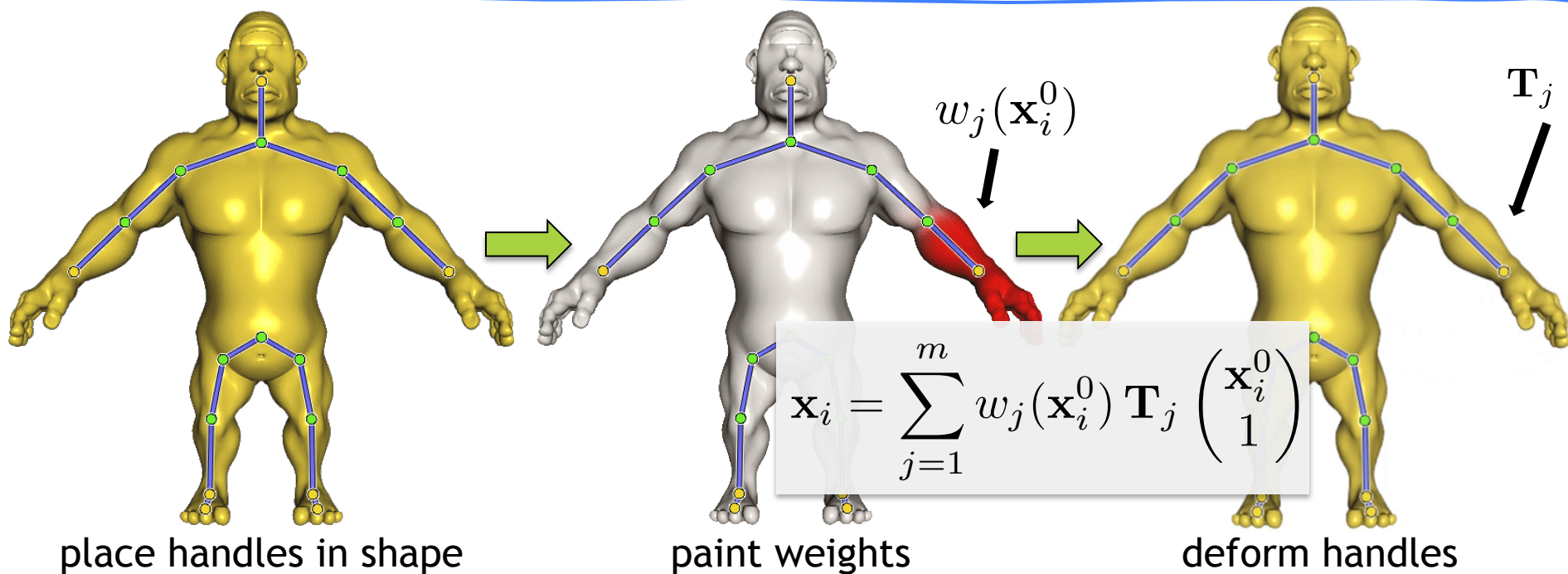




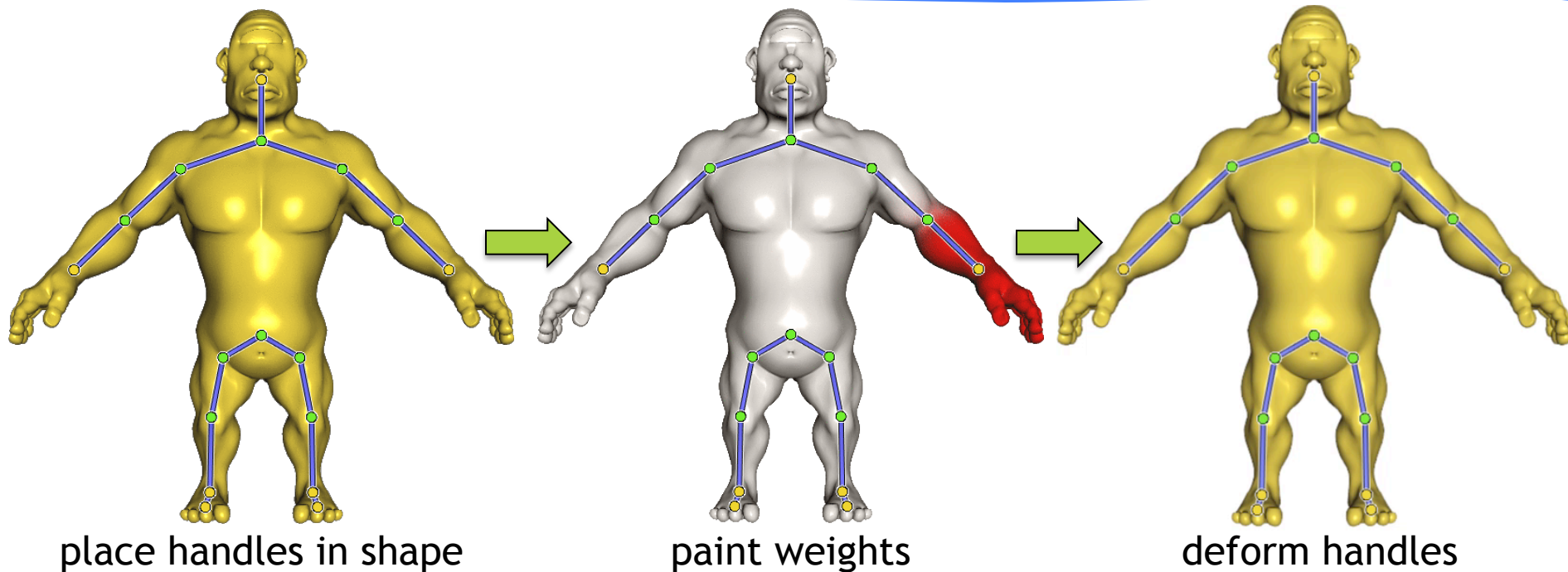
# Linear Blend Skinning rigging preferred for its real-time performance



# Linear Blend Skinning rigging preferred for its real-time performance



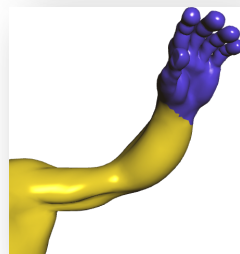
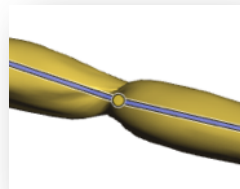
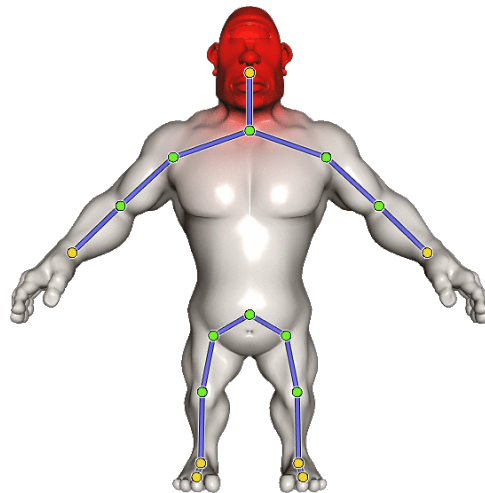
# Linear Blend Skinning rigging preferred for its real-time performance



# Challenges with LBS

- Weight functions  $w_j$ 
  - Need intuitive, general and automatic weights
- Degrees of freedom  $\mathbf{T}_j$ 
  - Let the energy decide!
- Richness of achievable deformations
  - Want to avoid common LBS pitfalls - candy wrapper, collapses

$$\mathbf{x}_i = \sum_{j=1}^m \underline{w_j(\mathbf{x}_i^0)} \underline{\mathbf{T}_j} \begin{pmatrix} \mathbf{x}_i^0 \\ 1 \end{pmatrix}$$

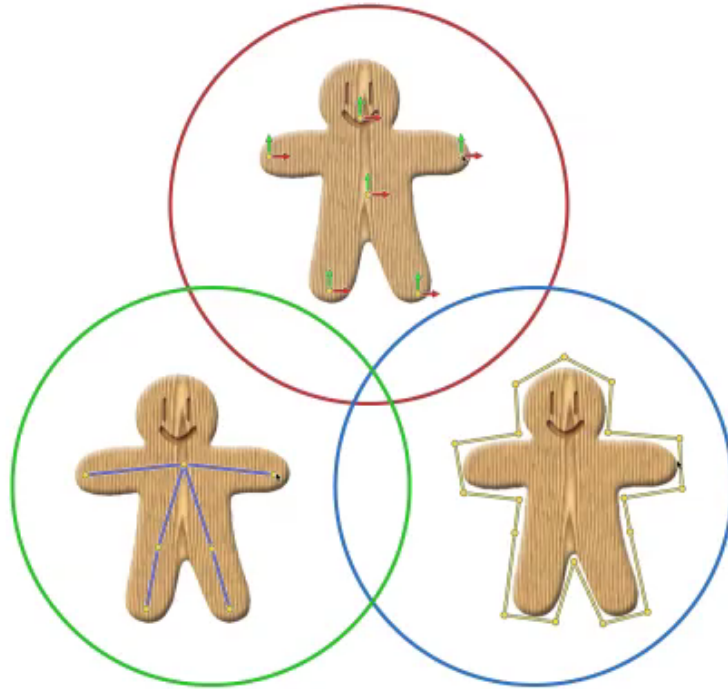


Alec Jacobson, Ilya Baran, Jovan Popović, S  
ACM SIGGRAPH 2011; selected for Research Highlights in CACM (2013)

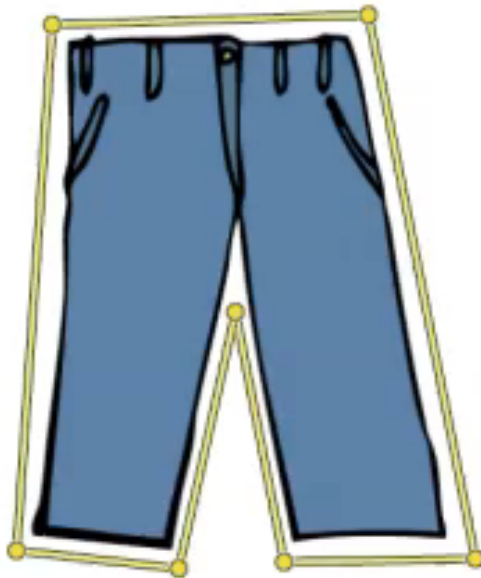
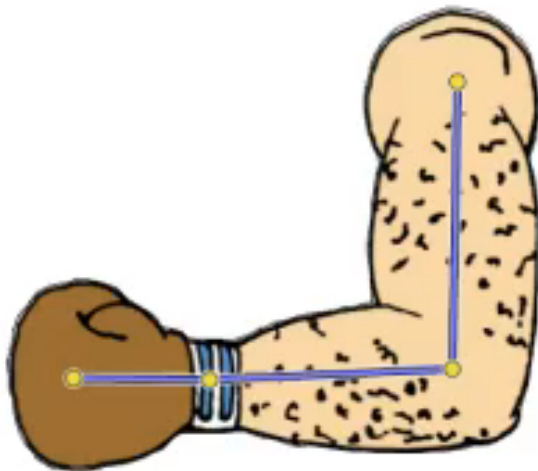
---

# Bounded Biharmonic Weights

# Automatic weights that unify points, skeletons and cages



# Weights should be smooth, shape-aware, positive and *intuitive*



# Weights must be smooth everywhere, *especially* at handles

---



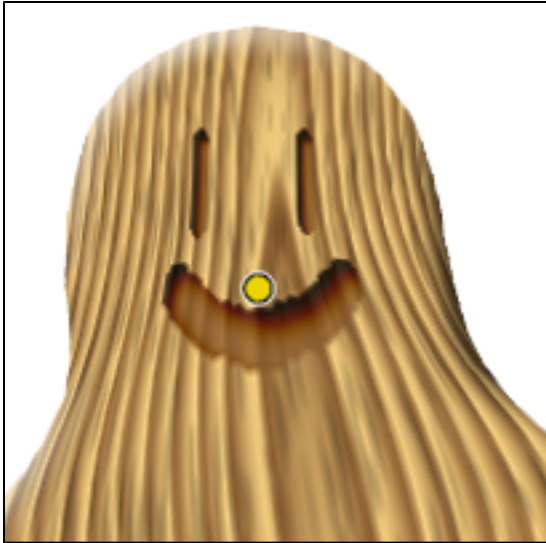
Bounded Biharmonic Weights



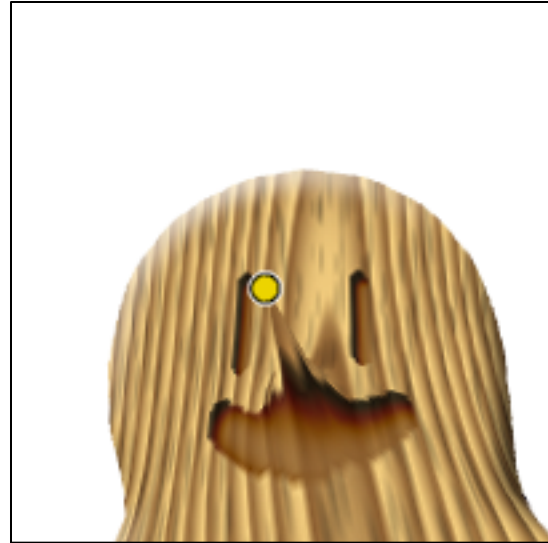
Extension of Harmonic Coordinates  
[Joshi et al. 2005]



# Weights must be smooth everywhere, *especially* at handles

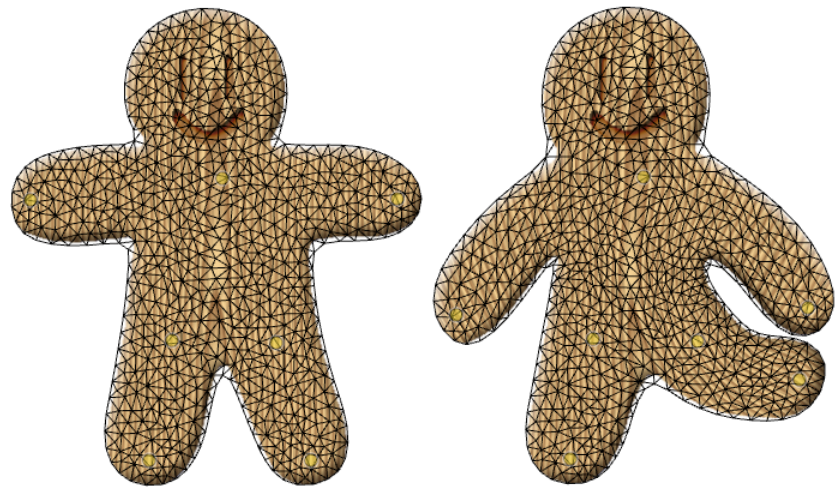


Bounded Biharmonic Weights

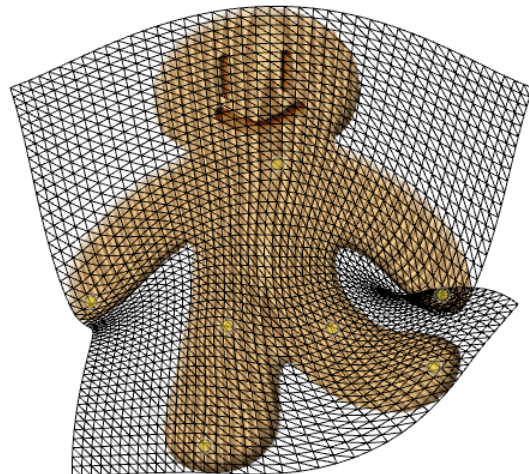


Extension of Harmonic Coordinates  
[Joshi et al. 2005]

# Shape-awareness ensures respect of domain's features



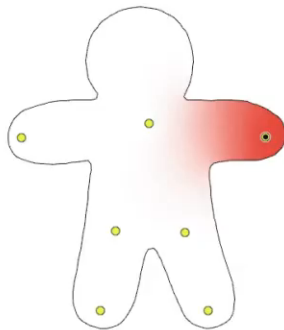
Bounded Biharmonic Weights



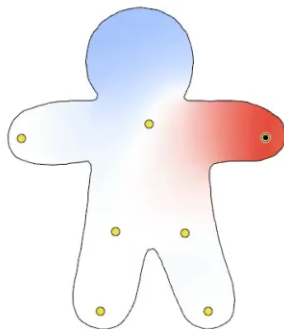
Non-shape-aware methods  
e.g. [Schaefer et al. 2006]

# Non-negative weights are necessary for intuitive response

Bounded Biharmonic Weights



Unconstrained biharmonic  
[Botsch and Kobbelt 2004]




# Weights must maintain other simple, but important properties

---

$$\sum_{j \in H} w_j(\mathbf{x}^0) = 1$$

Partition of unity

Handle vertices

$$w_j \Big|_{H_k} = \delta_{jk}$$


$w_j$  is linear along cage faces

Interpolation of handles

How about  $w_j(\mathbf{x}^0) = d(\mathbf{x}^0, H_j)^{-1}$  ?



# Inverse distance methods inherently suffer from *fall-off effect*

---

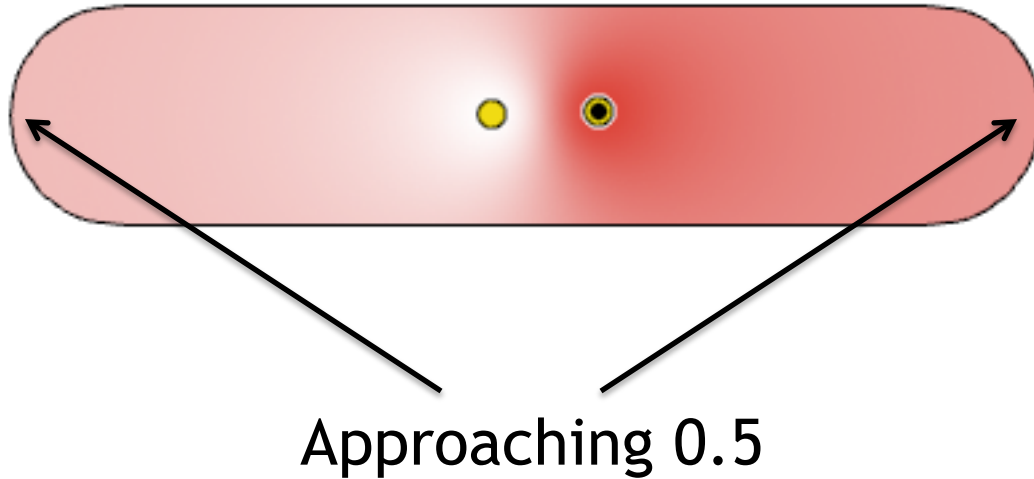


# Inverse distance methods inherently suffer from *fall-off effect*

---



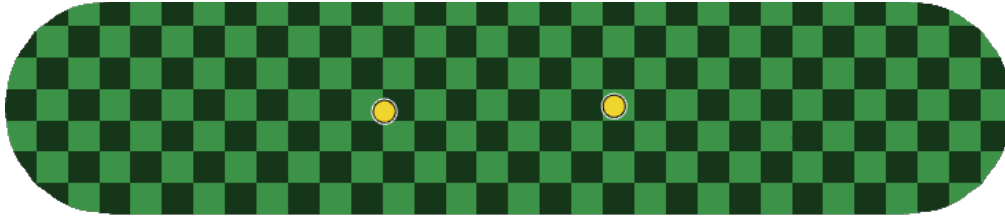
# Inverse distance methods inherently suffer from *fall-off effect*



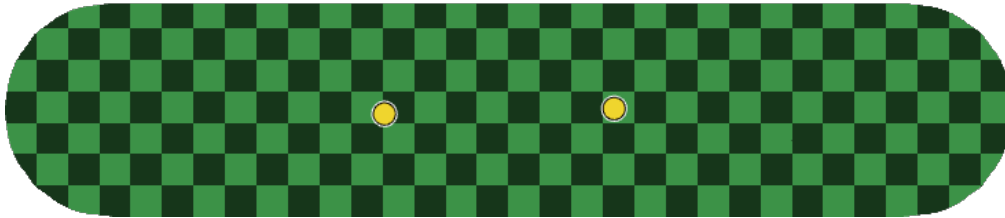


# Inverse distance methods inherently suffer from *fall-off effect*

Inverse-distance weights



BBW



# *Bounded biharmonic weights* enforce properties as constraints to minimization

---

$$\arg \min_{w_j} \frac{1}{2} \int_{\Omega} |\Delta w_j|^2 dV$$

$$w_j \Big|_{H_k} = \delta_{jk}$$

$w_j$  is linear along cage faces

# Bounded biharmonic weights enforce properties as constraints to minimization

$$\arg \min_{w_j} \frac{1}{2} \int_{\Omega} |\Delta w_j|^2 dV$$

$$w_j \Big|_{H_k} = \delta_{jk}$$

$w_j$  is linear along cage faces

Constant inequality constraints

$$0 \leq w_j(\mathbf{x}^0) \leq 1$$

Partition of unity

$$\sum_{j \in H} w_j(\mathbf{x}^0) = 1$$

# Bounded biharmonic weights enforce properties as constraints to minimization

$$\arg \min_{w_j} \frac{1}{2} \int_{\Omega} |\Delta w_j|^2 dV$$

$$w_j \Big|_{H_k} = \delta_{jk}$$

$w_j$  is linear along cage faces

Constant inequality constraints

$$0 \leq w_j(\mathbf{x}^0) \leq 1$$

Solve independently and  
normalize

$$w_j(\mathbf{x}^0) = \frac{w_j(\mathbf{x}^0)}{\sum_{i \in H} w_i(\mathbf{x}^0)}$$

# Weights optimized as precomputation at bind-time

$$\arg \min_{w_j} \frac{1}{2} \int_{\Omega} |\Delta w_j|^2 dV$$
$$w_j|_{H_k} = \delta_{jk}$$

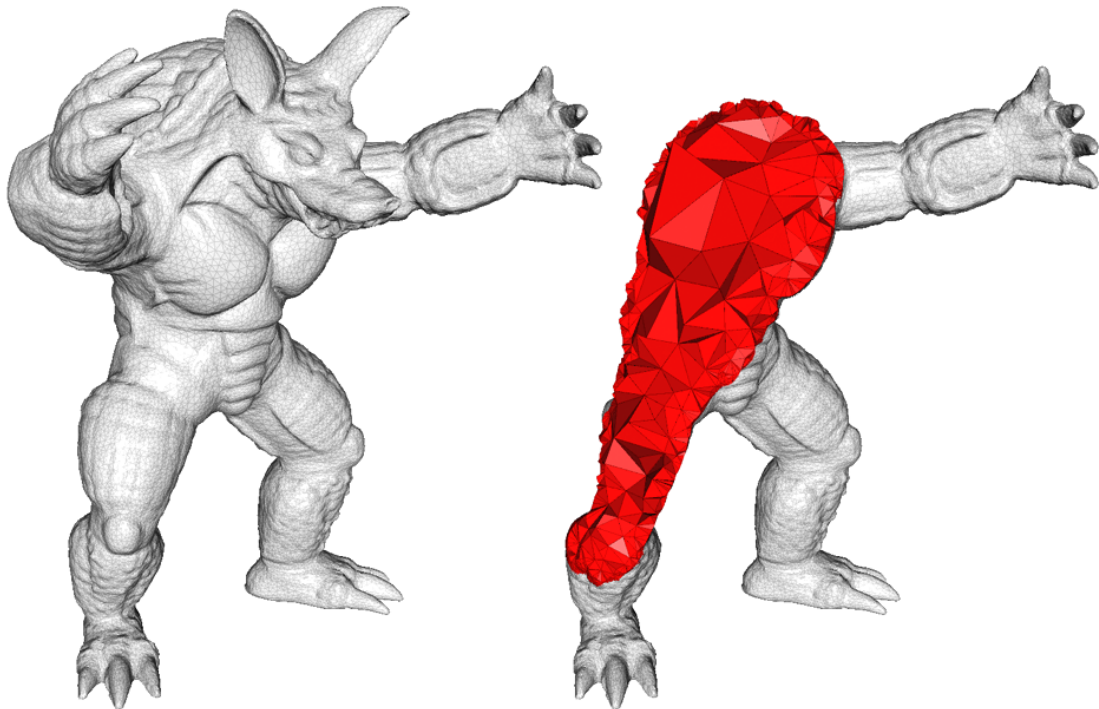
$w_j$  is linear along cage faces

$$0 \leq w_j(\mathbf{x}^0) \leq 1$$

FEM discretization

2D  $\rightarrow$  Triangle mesh

3D  $\rightarrow$  Tet mesh



# Weights optimized as precomputation at bind-time

$$\arg \min_{w_j} \frac{1}{2} \int_{\Omega} |\Delta w_j|^2 dV$$

$$w_j|_{H_k} = \delta_{jk}$$

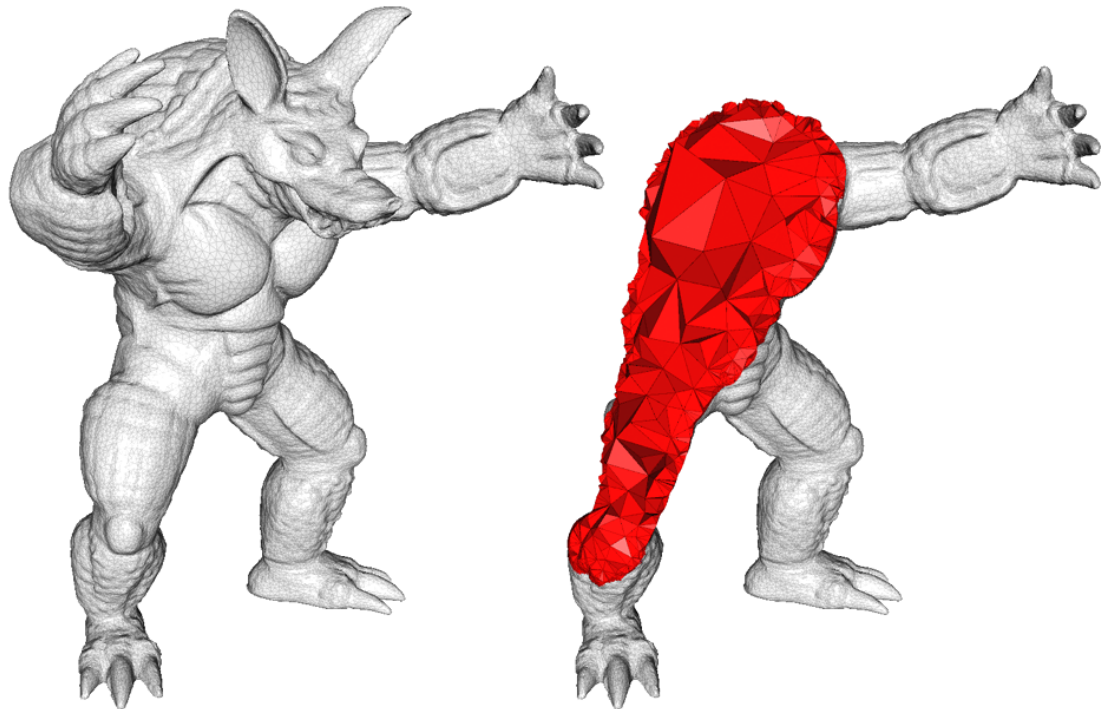
$w_j$  is linear along cage faces

$$0 \leq w_j(\mathbf{x}^0) \leq 1$$

**Sparse quadratic programming** with  
constant inequality constraints

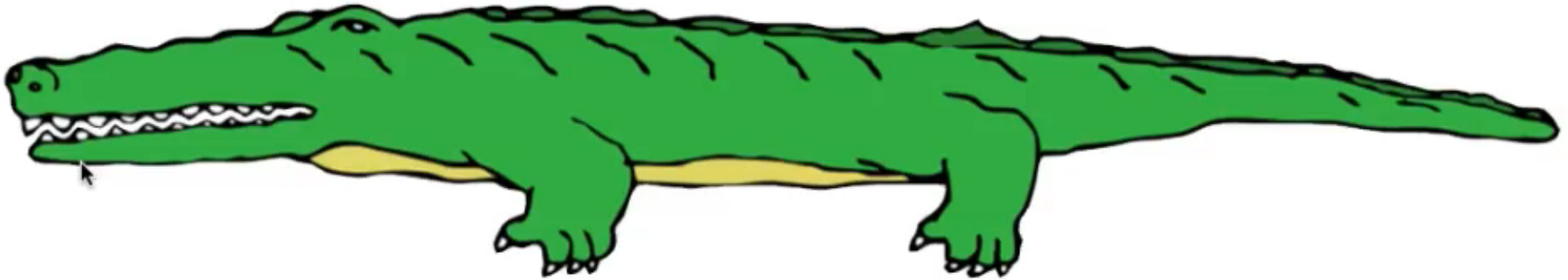
2D  $\rightarrow$  less than second per handle

3D  $\rightarrow$  tens of seconds per handle

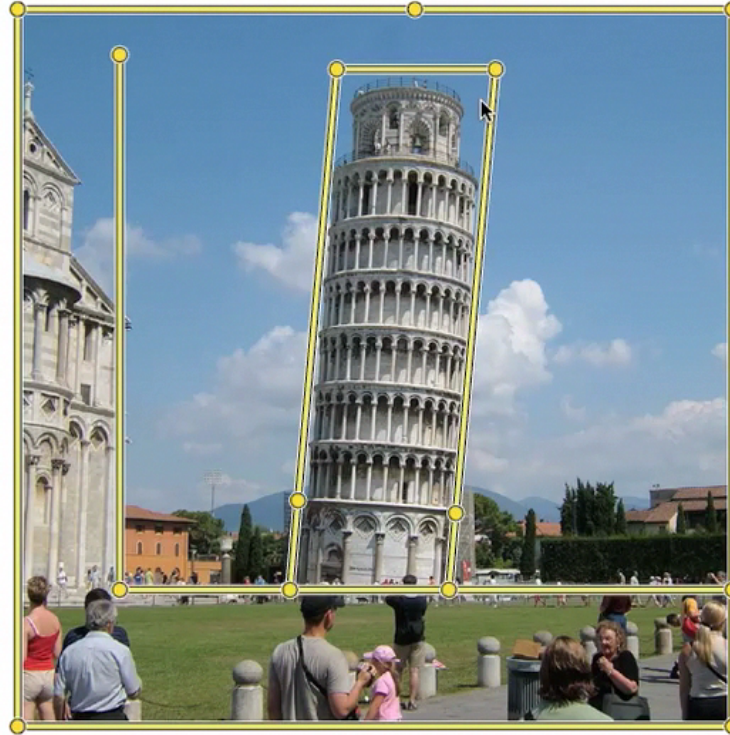


# Some examples of BBW in action

---

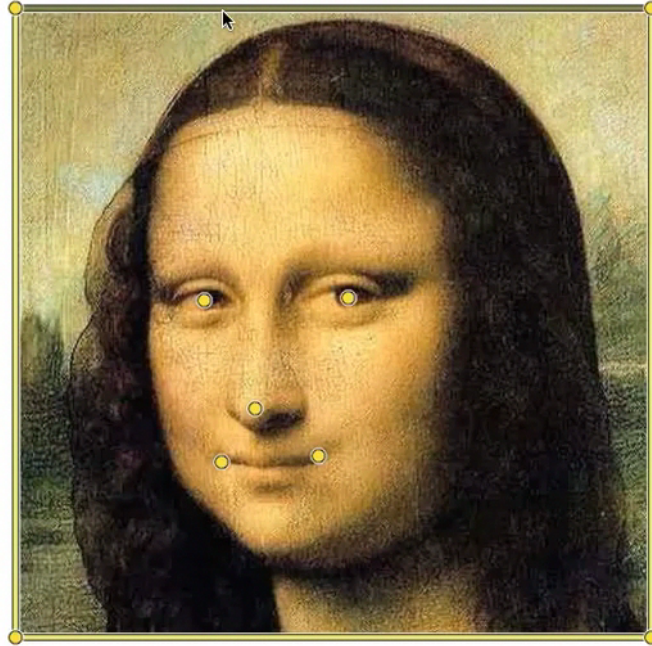


# Some examples of BBW in action





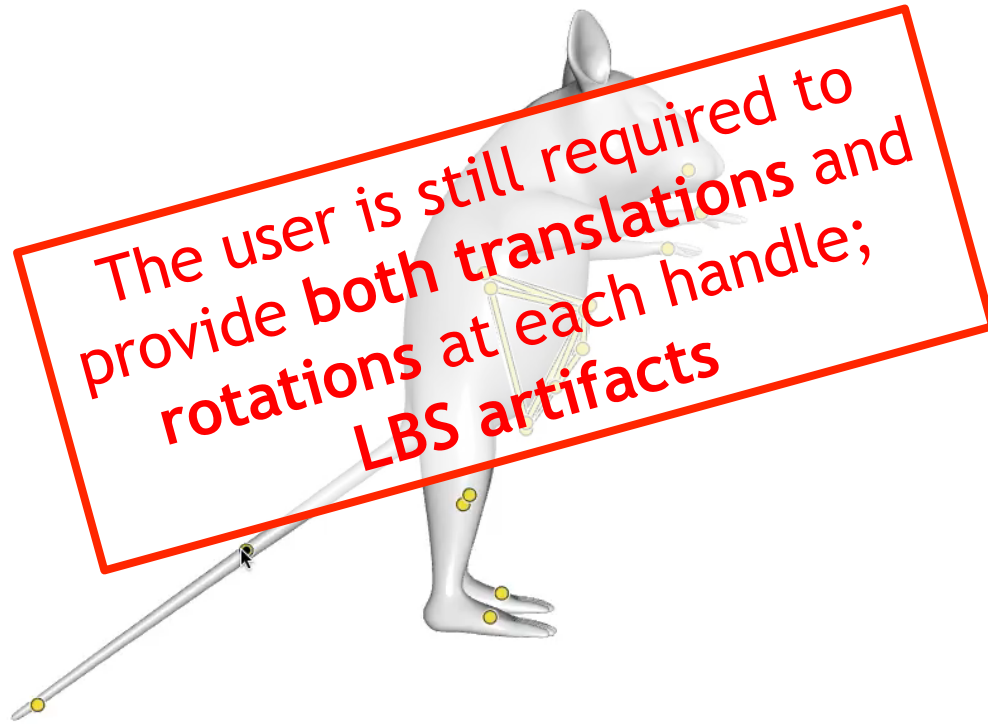
# Some examples of BBW in action



# 3D Characters



# Mixing different handle types



Alec Jacobson, Ilya Baran, Ladislav Kavan, Jovan Popović, S  
ACM SIGGRAPH 2012

---

# Fast Automatic Skinning Transformations

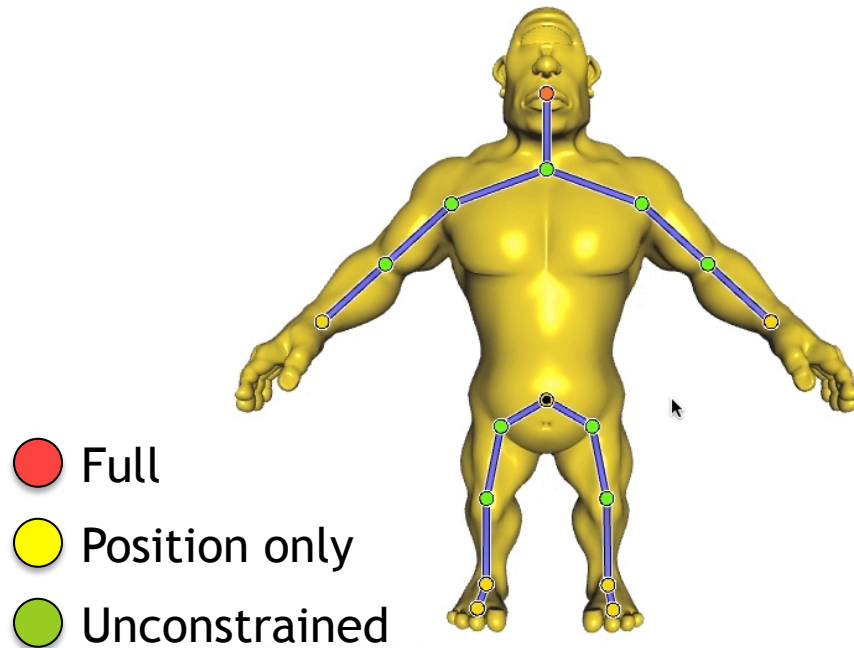
# User specifies subset of parameters, automatically optimize remaining ones

Reduced optimization

$$\arg \min_{\mathbf{T}} E(\mathbf{M}\mathbf{T})$$

User constraints

$$\underbrace{\begin{bmatrix} \mathbf{I}_{\text{full}} \\ \mathbf{M}_{\text{pos}} \end{bmatrix}}_{\mathbf{M}_{\text{eq}}} \mathbf{T} = \underbrace{\begin{bmatrix} \mathbf{T}_{\text{full}} \\ \mathbf{P}_{\text{pos}} \end{bmatrix}}_{\mathbf{P}_{\text{eq}}}$$



# User specifies subset of parameters, automatically optimize remaining ones

Full optimization

$$\arg \min_{\mathbf{x}} E(\mathbf{x})$$

Nonlinear elastic  
energy of the shape

Reduced model

$$\mathbf{x}_i = \sum_{j=1}^m w_j(\mathbf{x}_i^0) \mathbf{T}_j \begin{pmatrix} \mathbf{x}_i^0 \\ 1 \end{pmatrix}$$

Matrix form

$$\mathbf{x} = \mathbf{MT}$$

Reduced optimization

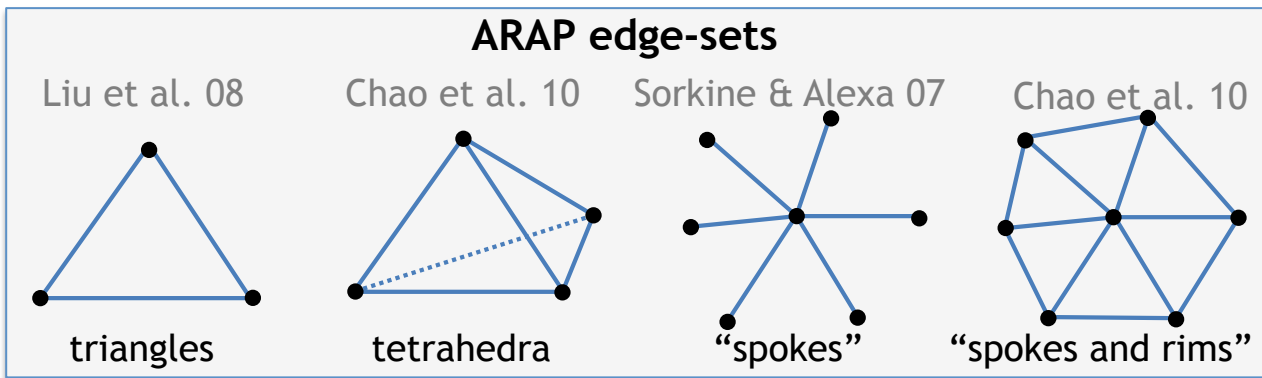
$$\arg \min_{\mathbf{T}} E(\mathbf{MT})$$

**E is nonlinear;  
evaluation still  
expensive**

# We reduce any *as-rigid-as-possible* energies

Full energy  $E(\mathbf{x}) = \sum_{k \in \text{Cells}} \sum_{(i,j) \in \mathcal{E}_k} c_{ijk} \|(\mathbf{x}_i - \mathbf{x}_j) - \mathbf{R}_k(\mathbf{x})(\mathbf{x}_i^0 - \mathbf{x}_j^0)\|^2$

Best rotation that aligns deformed configuration  $\mathcal{E}_k$  to rest-pose



# We reduce any *as-rigid-as-possible* energies

Full energy  $E(\mathbf{x}) = \sum_{k \in \text{Cells}} \sum_{(i,j) \in \mathcal{E}_k} c_{ijk} \|(\mathbf{x}_i - \mathbf{x}_j) - \mathbf{R}_k(\mathbf{x})(\mathbf{x}_i^0 - \mathbf{x}_j^0)\|^2$

Best rotation that aligns deformed configuration  $\mathcal{E}_k$  to rest-pose

Reduction to LBS  
subspace:

$$\mathbf{x} = \mathbf{M}\mathbf{T}$$

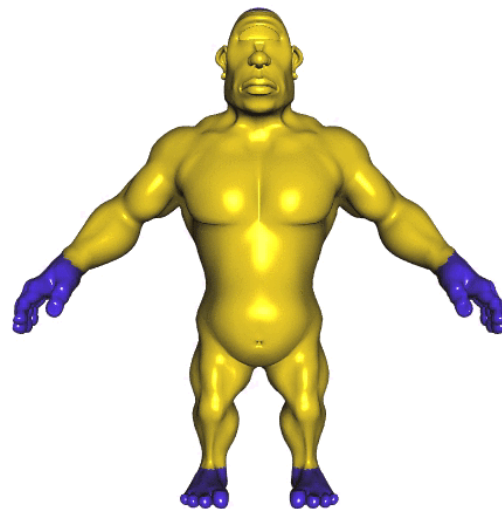
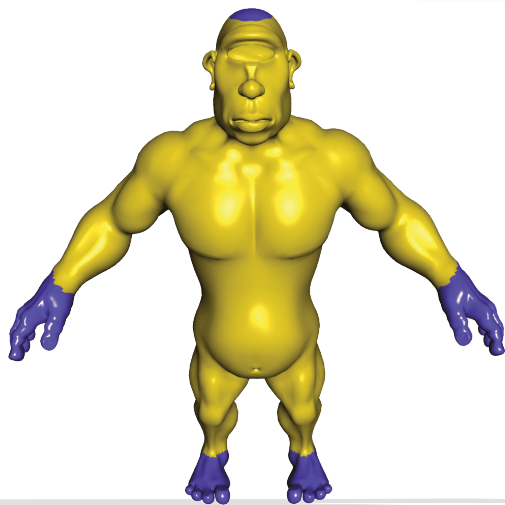
**Problem: still  
many SVDs to do!**

$$\#\{\mathbf{R}_k(\mathbf{x})\} = O(\#\text{vertices})$$



# Rotation evaluations may be reduced by clustering in *weight space*

$$E(\mathbf{x}) = \sum_{k \in \text{Cells}} \sum_{(i,j) \in \mathcal{E}_k} c_{ijk} \|(\mathbf{x}_i - \mathbf{x}_j) - \mathbf{R}_k(\mathbf{x})(\mathbf{x}_i^0 - \mathbf{x}_j^0)\|^2$$



# Rotation evaluations may be reduced by clustering in *weight space*

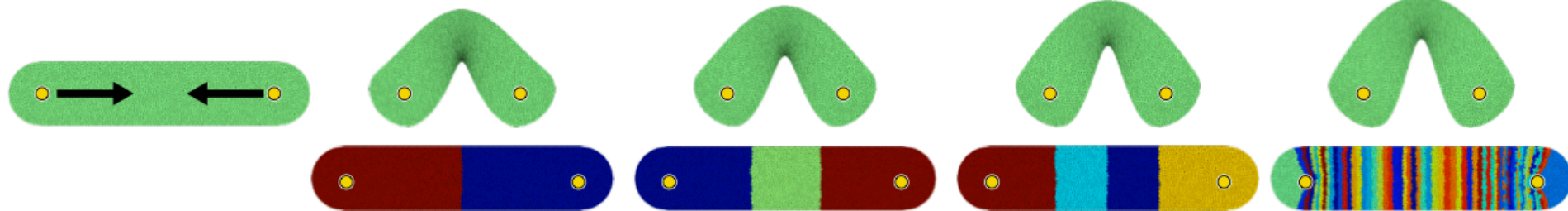
$$E(\mathbf{x}) = \sum_{k \in \text{Cells}} \sum_{(i,j) \in \mathcal{E}_k} c_{ijk} \|(\mathbf{x}_i - \mathbf{x}_j) - \mathbf{R}_k(\mathbf{x})(\mathbf{x}_i^0 - \mathbf{x}_j^0)\|^2$$

#Cells = 2

#Cells = 3

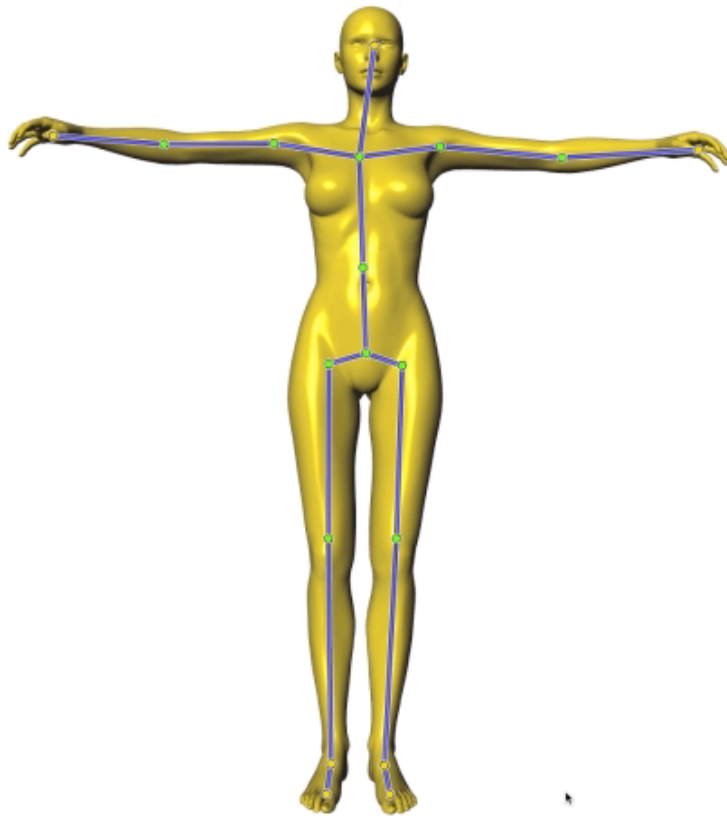
#Cells = 4

#Cells = 64



# Real-time automatic degrees of freedom

---



With more and more user constraints  
we fall back to standard skinning

---



# Extra weights would expand subspace to approximate elasticity better

---

$$\mathbf{x}_i = \sum_{j=1}^m w_j(\mathbf{x}_i^0) \mathbf{T}_j \begin{pmatrix} \mathbf{x}_i^0 \\ 1 \end{pmatrix}$$

$$\mathbf{x} = \mathbf{MT}$$

# Extra weights would expand subspace to approximate elasticity better

---

$$\mathbf{x}_i = \sum_{j=1}^m w_j(\mathbf{x}_i^0) \mathbf{T}_j \begin{pmatrix} \mathbf{x}_i^0 \\ 1 \end{pmatrix} + \sum_{k=1}^{m_{\text{extra}}} \tilde{w}_k(\mathbf{x}_i^0) \mathbf{T}_k \begin{pmatrix} \mathbf{x}_i^0 \\ 1 \end{pmatrix}$$

$$\mathbf{X} = \mathbf{MT} + \mathbf{M}_{\text{extra}}\mathbf{T}_{\text{extra}}$$

# Extra weights would expand subspace to approximate elasticity better

## Need:

- smooth
- local, sparse
- respect intent of original weights

$$\mathbf{x}_i = \sum_{j=1}^m w_j(\mathbf{x}_i^0) \mathbf{T}_j \begin{pmatrix} \mathbf{x}_i^0 \\ 1 \end{pmatrix} + \sum_{k=1}^{m_{\text{extra}}} \tilde{w}_k(\mathbf{x}_i^0) \mathbf{T}_k \begin{pmatrix} \mathbf{x}_i^0 \\ 1 \end{pmatrix}$$

## Do not need:

- partition of unity
- interpolation
- scale, sign, etc.

$$\mathbf{X} = \mathbf{MT} + \mathbf{M}_{\text{extra}} \mathbf{T}_{\text{extra}}$$

# Extra weights expand deformation subspace, while respecting user intent

## Computation:

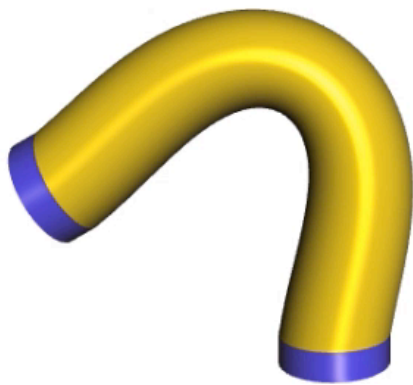
- Distribute samples in weight space
- Smooth B-Spline “bumps” around samples





# Subspace now rich enough for lightning fast **variational elastic modeling**

---



Reference nonlinear deformation:  
PriMo [Botsch et al. SGP 2006]

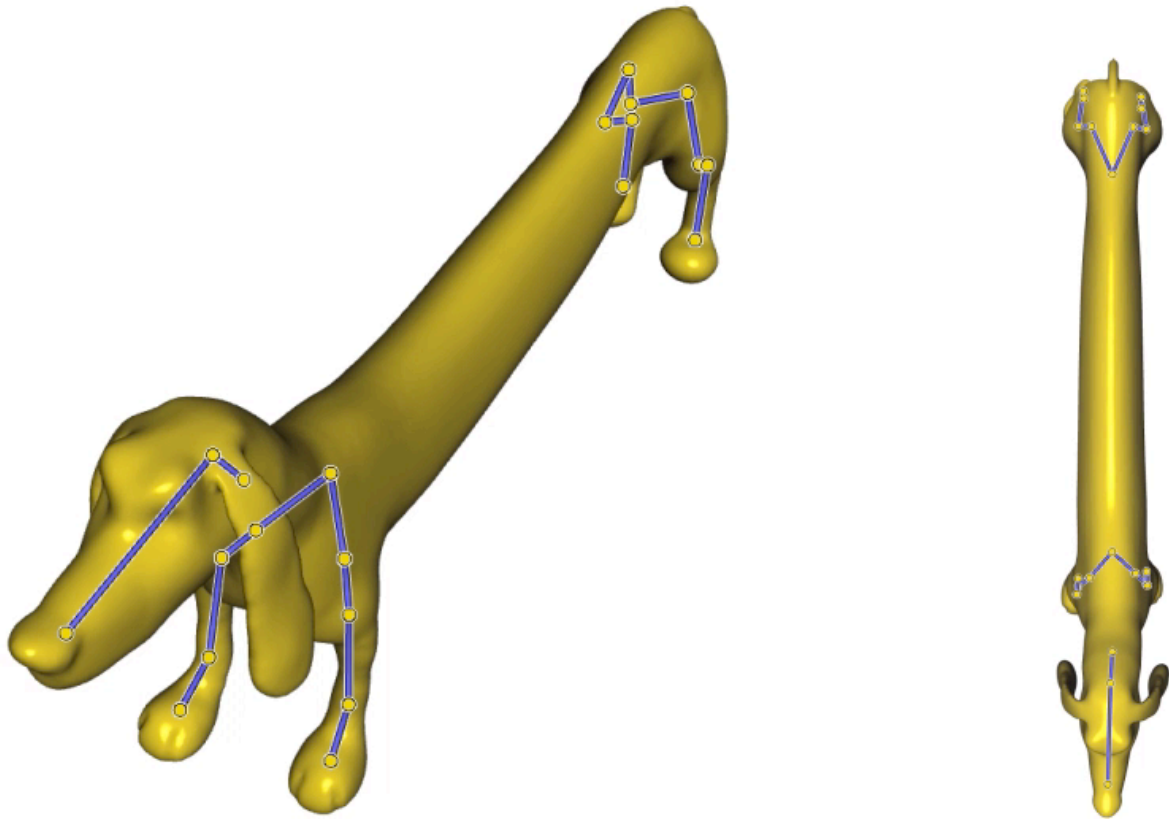


Our reduced method

# Extra weights and disjoint skeletons make flexible control easy

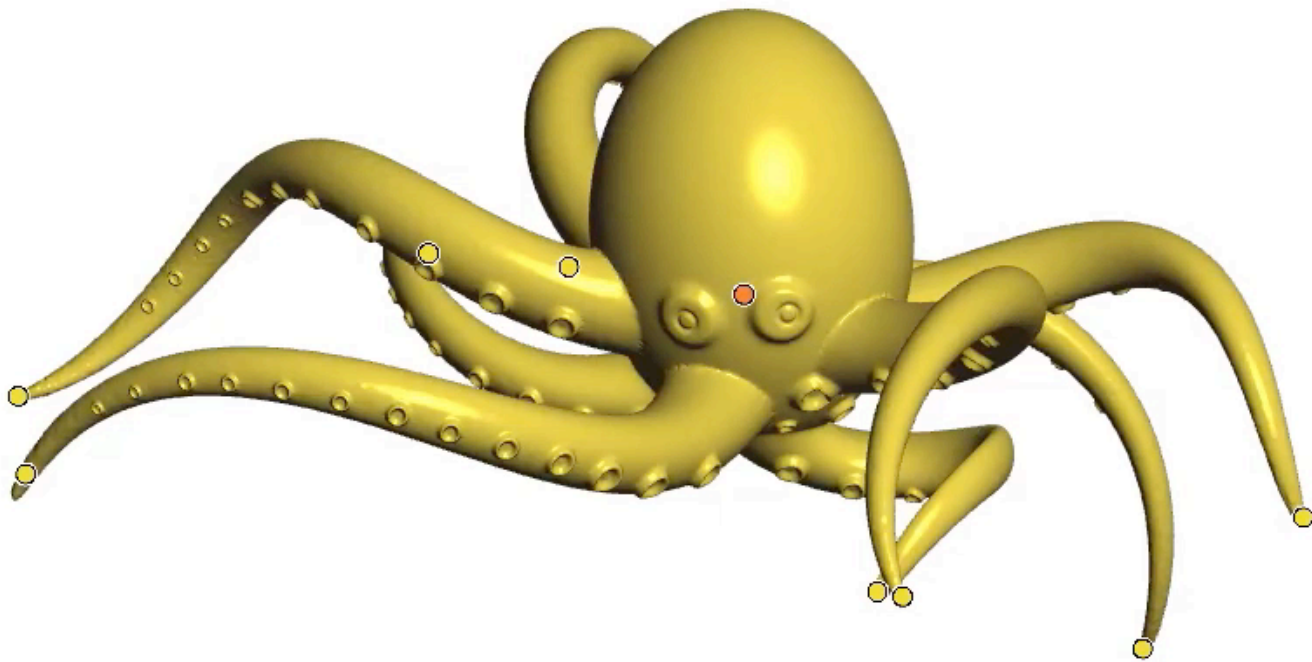
---

Shape-aware IK!

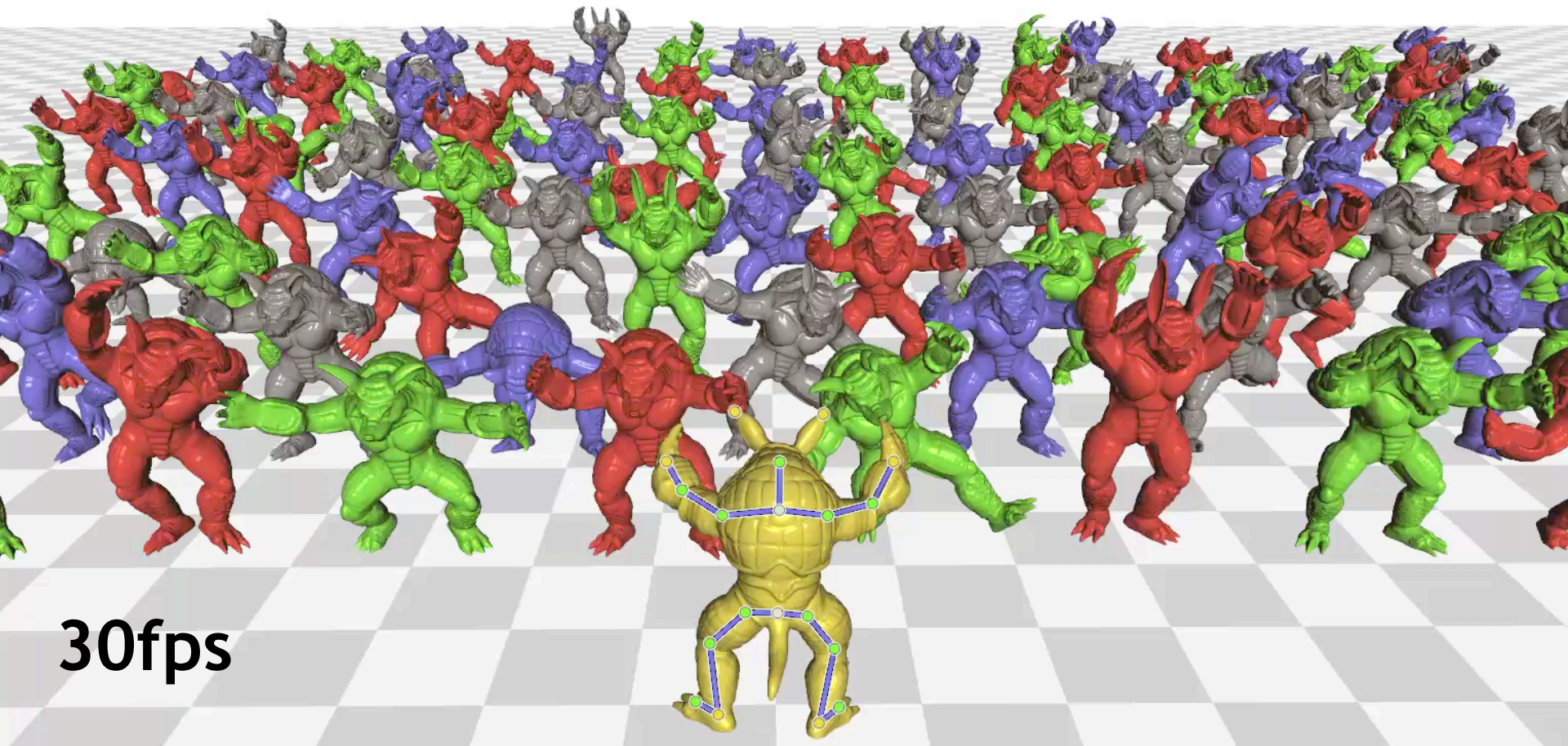


# Simple drag-only interface for point handles

---



# 100 Armadillos, 86K triangles each



30fps

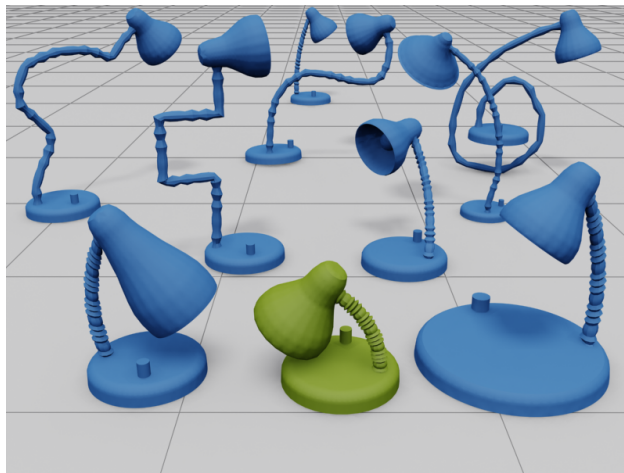
Realtime variational mesh editing solved...

---

# What next??

# Non-Elastic Deformations

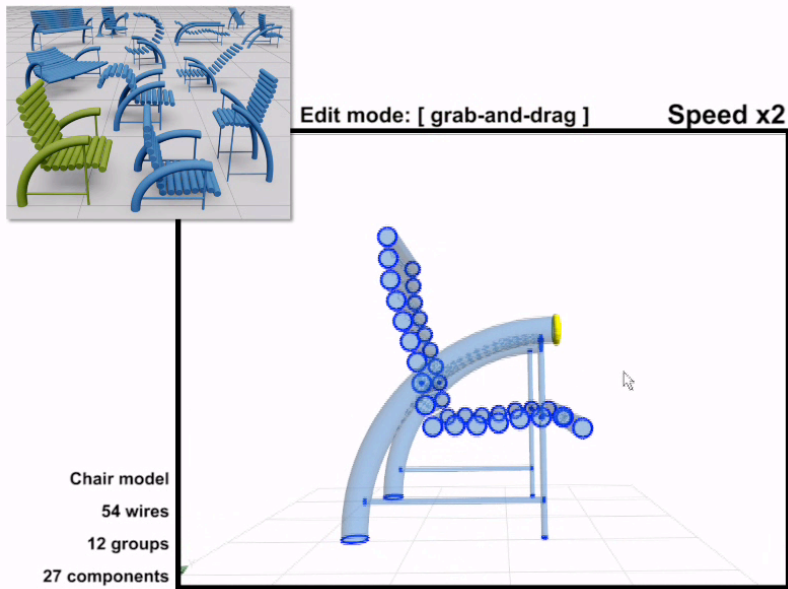
- Many shapes, e.g. man-made, are **not** made of rubber
  - Extract and preserve high-level structures while editing!



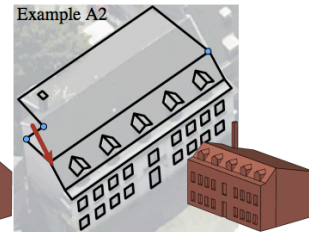
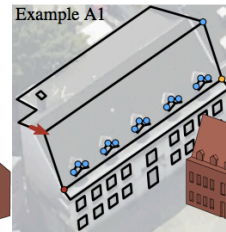
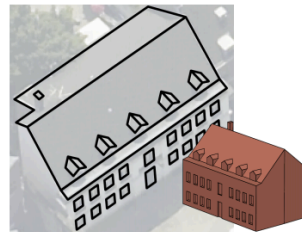
Images from **iWires** [Gal, S, Mitra, Cohen-Or, SIGGRAPH 2009]



# Non-Elastic Deformations



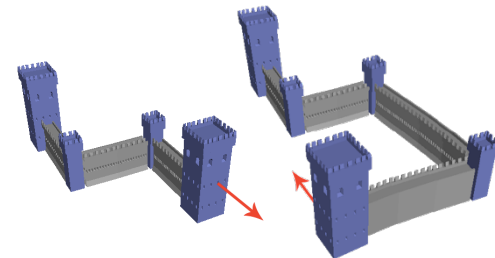
**iWires** [Gal, S, Mitra, Cohen-Or 2009]  
**Sequels**, e.g. "Component-wise controllers for structure-preserving shape manipulation", EG 2011



Habbecke & Kobbelt, EG 2012



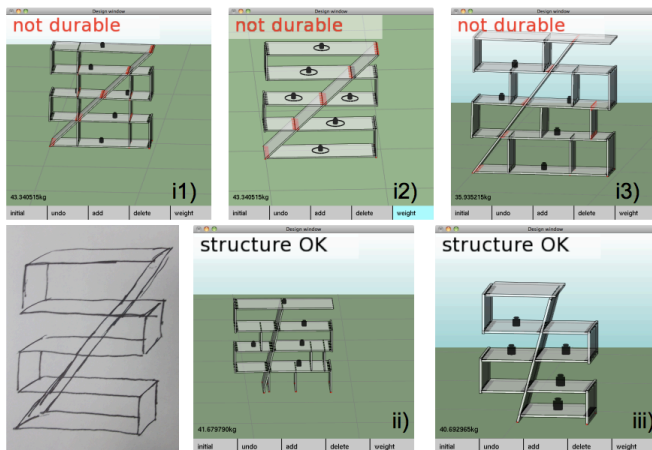
Bokeloh et al.  
SIGGRAPH 2012



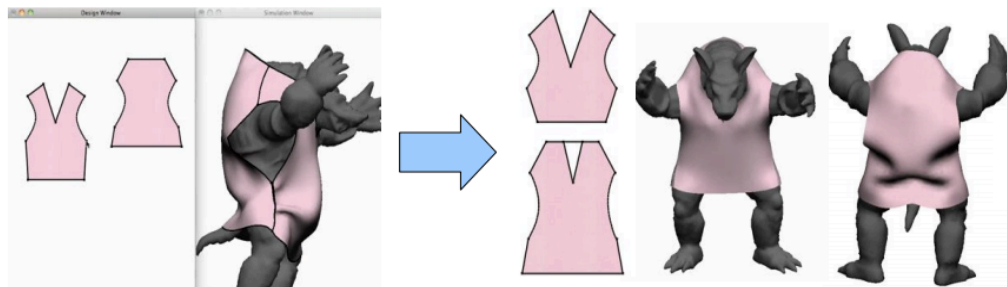
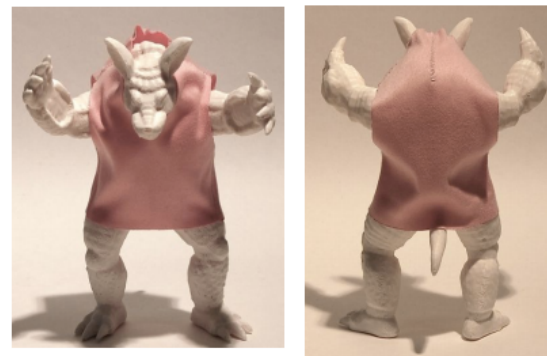
Milliez et al.  
EG 2013

# Modeling for the Real World

- From modeling directly to manufacturing
- Need to find the right **balance** between **physical constraints** and **artistic freedom**



Umetani et al. SIGGRAPH 2012



Umetani et al. SIGGRAPH 2011



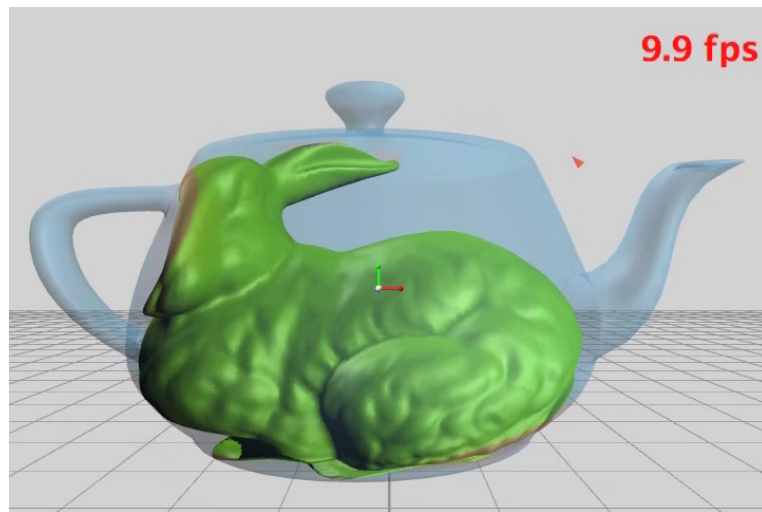
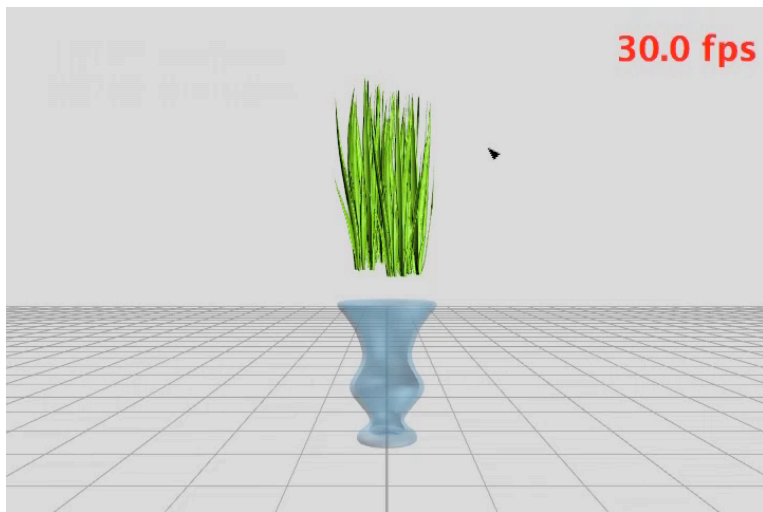
# Modeling for the Real World

---

- Specialized systems vs. general principles?

# Modeling for the Real World

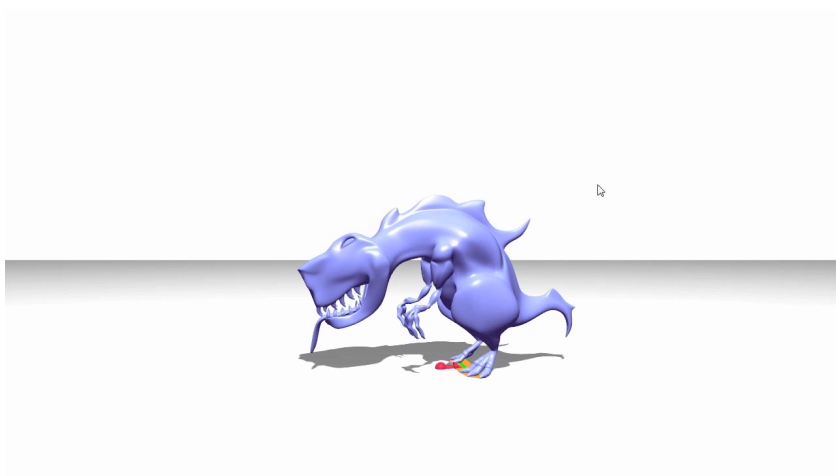
- Specialized systems vs. general principles?
- Example: **Self-intersections and collisions**



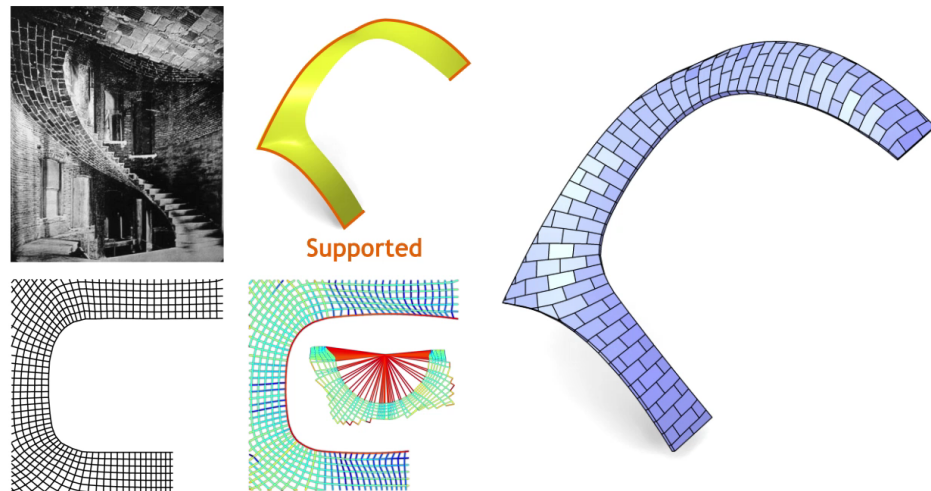
“Interface Aware Geometric Modeling”, Harmon, Panozzo, S, Zorin, SIGGRAPH ASIA 2011

# Modeling for the Real World

- Specialized systems vs. general principles?
- Example: **Gravity**



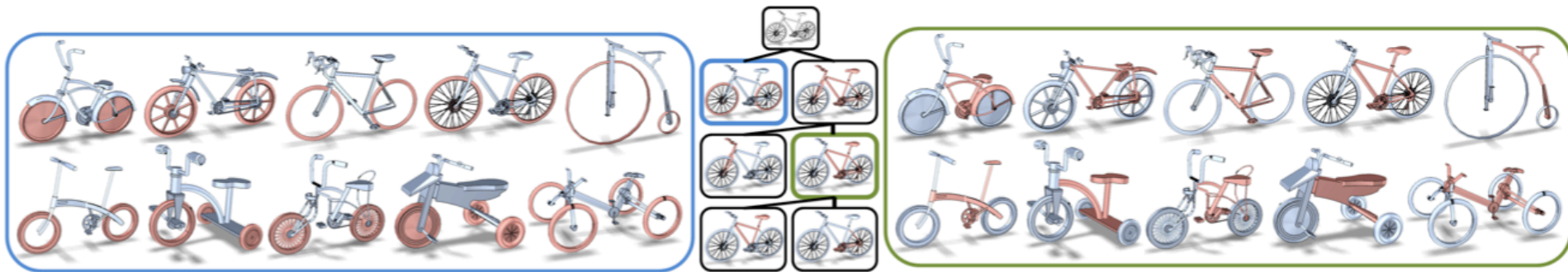
“Make It Stand”, Prevost, Whiting, Lefebvre, S  
SIGGRAPH 2013



“Designing Masonry Models”, Panozzo, Block, S  
SIGGRAPH 2013

# Big Data

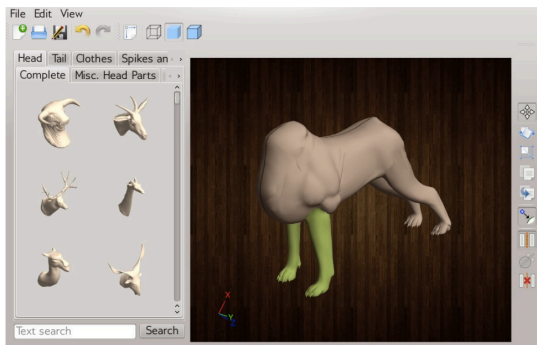
- Large model collections: learn **model structure**, **semantic segmentation**, inspiration for modeling



“Co-Hierarchical Analysis of Shape Structures”, van Kaick et al., SIGGRAPH 2013

# Big Data

- Large model collections: learn model structure, semantic segmentation, inspiration for modeling



“Probabilistic Reasoning for Assembly-Based 3D Modeling”, Chaudhuri et al., SIGGRAPH 2011

- Not just static data but **modeling process data?**

Thank You!

A decorative blue wavy line that spans the width of the slide, positioned below the 'Thank You!' text.