Scene Graphs

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CS 4810: Graphics

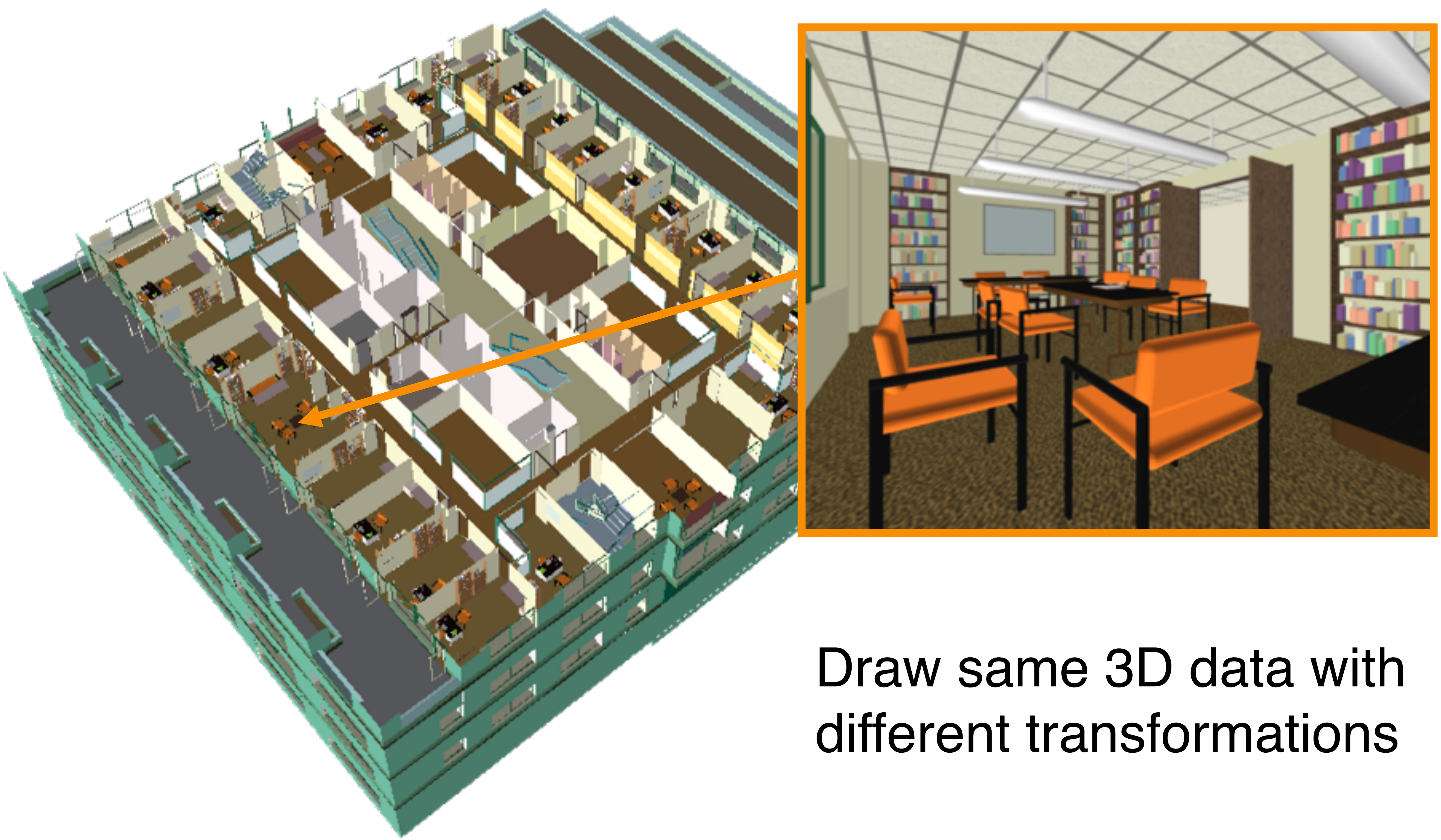
Acknowledgment: slides by Jason Lawrence, Misha Kazhdan, Allison Klein, Tom Funkhouser, Adam Finkelstein and David Dobkin

Overview

- 2D Transformations
  - Basic 2D transformations
  - Matrix representation
  - Matrix composition
- 3D Transformations
  - Basic 3D transformations
  - Same as 2D
- Transformation Hierarchies
  - Scene graphs
  - Ray casting

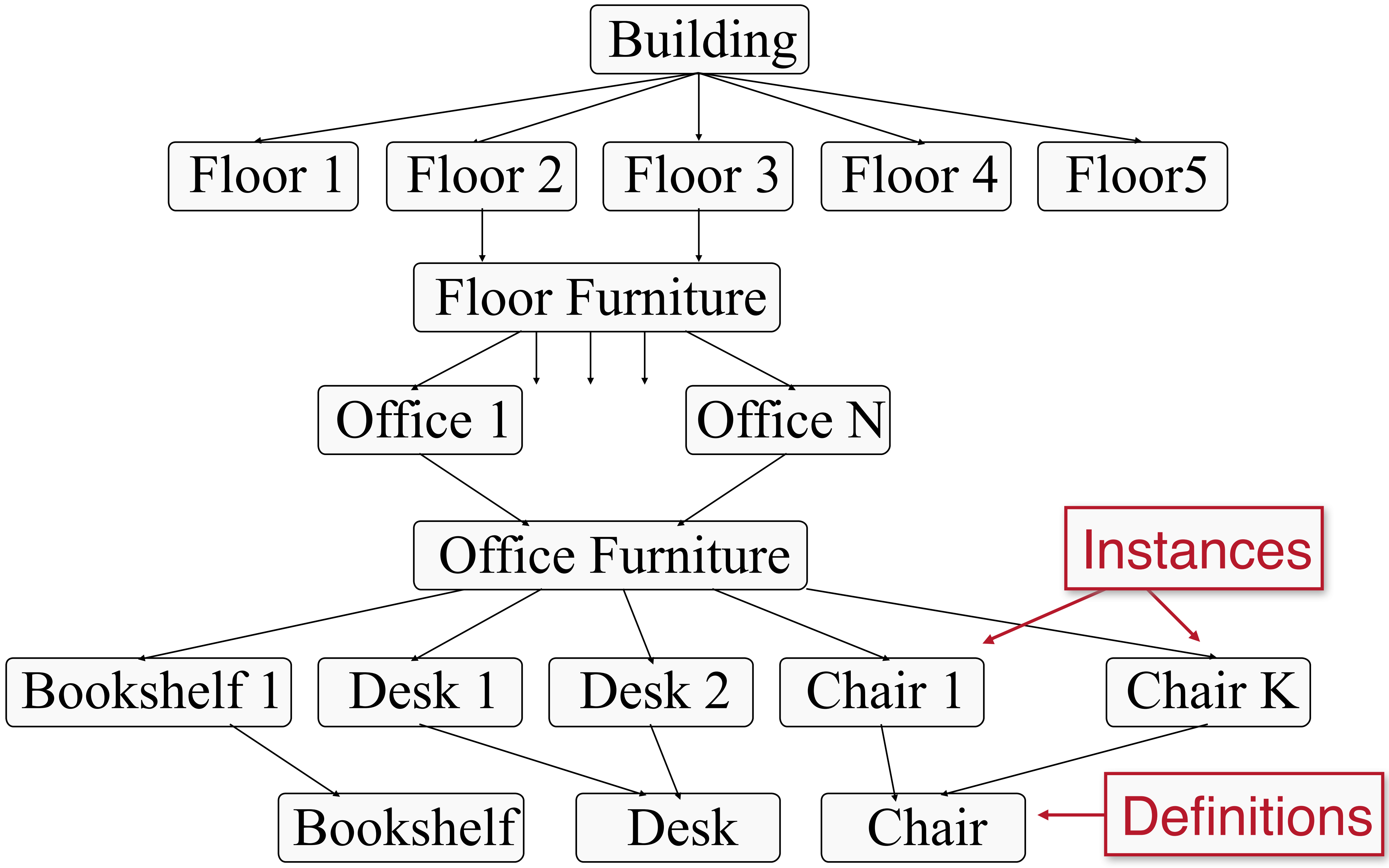
Transformation Example 1

- An object may appear in a scene multiple times



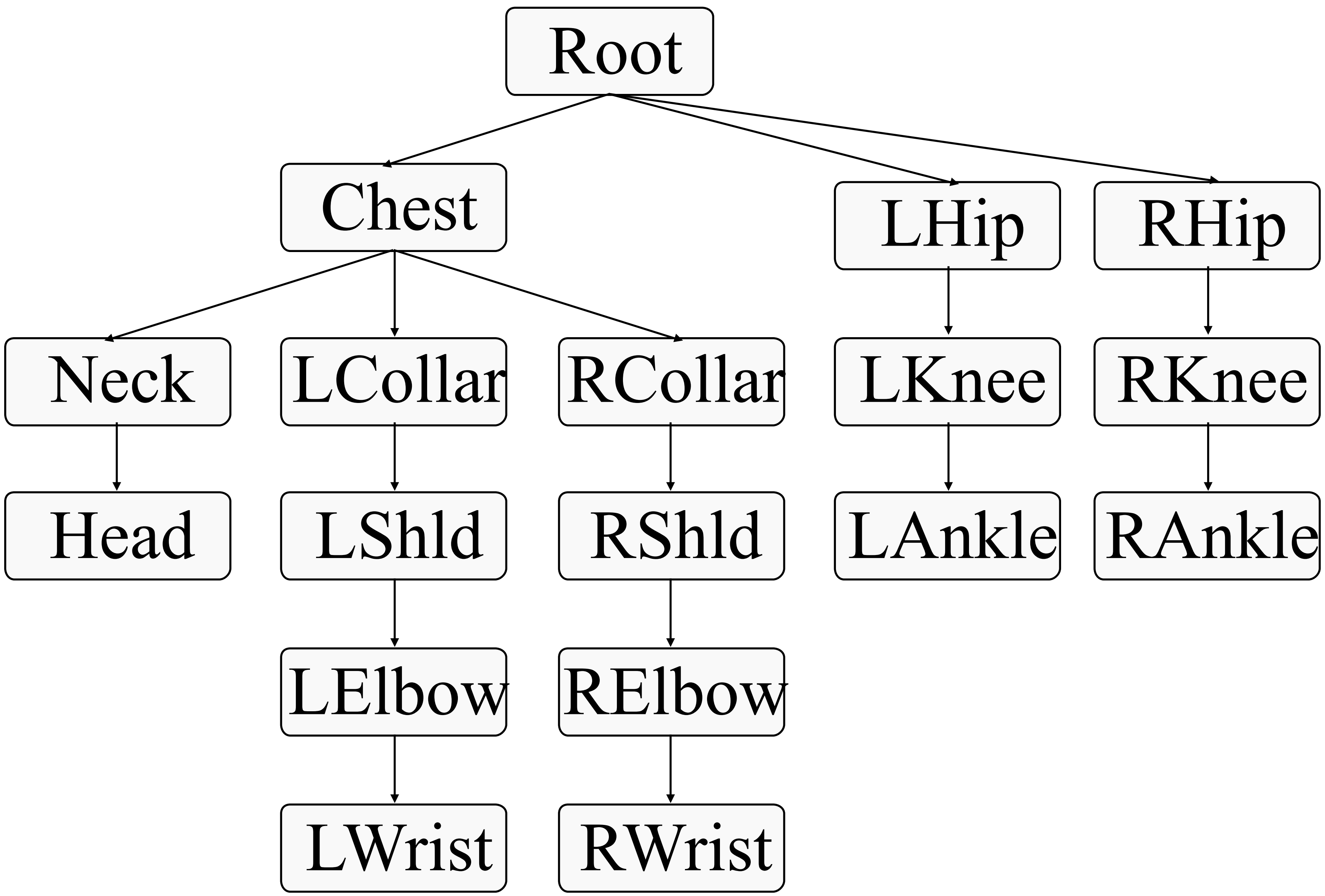
Draw same 3D data with different transformations

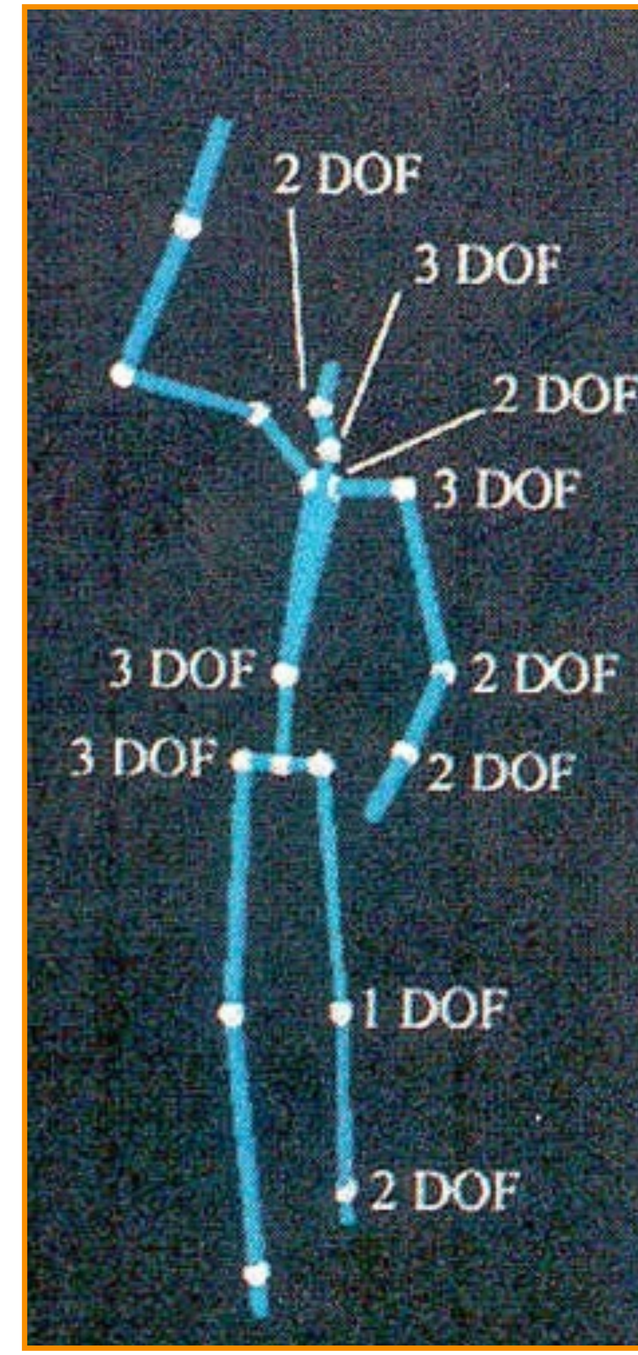
Transformation Example 1



Transformation Example 2

- Well-suited for humanoid characters





Rose et al. `96

Scene Graphs

- Allow us to have multiple instances of a single model – providing a reduction in model storage size
- Allow us to model objects in local coordinates and then place them into a global frame – particularly important for animation

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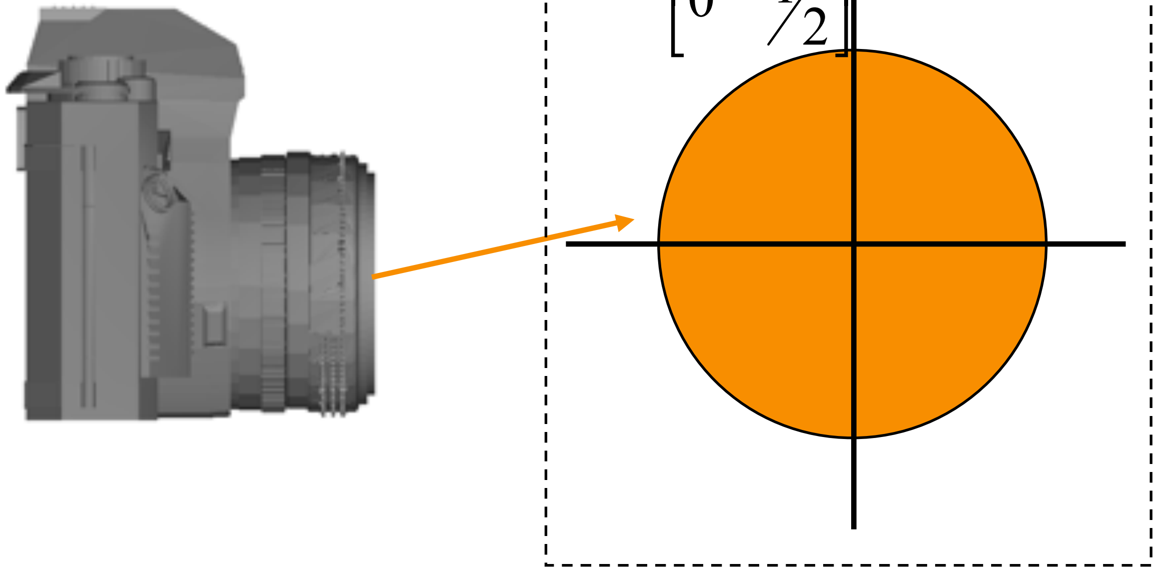
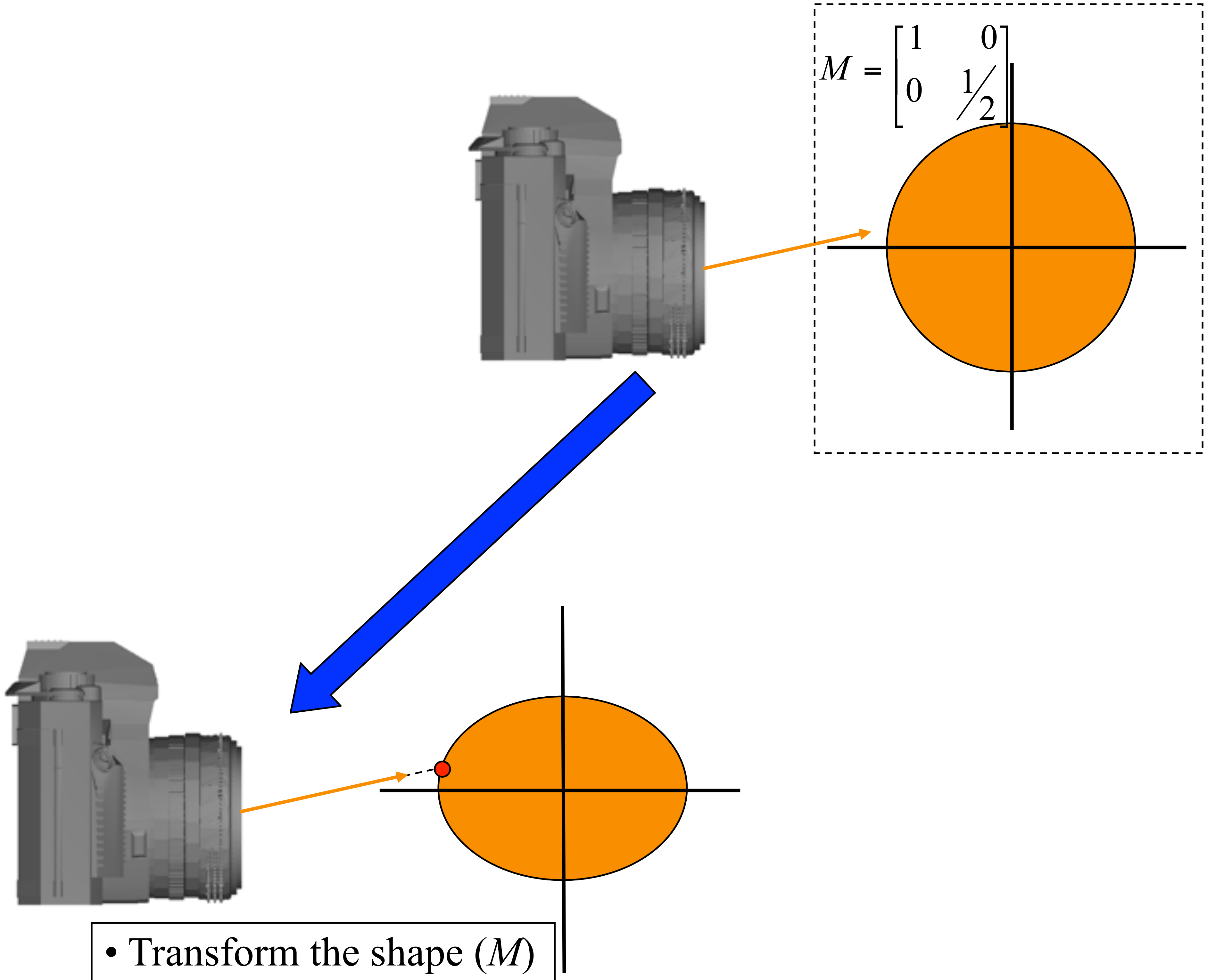
- Allow us to have multiple instances of a single model – providing a reduction in model storage size
- Allow us to model objects in local coordinates and then place them into a global frame – particularly important for animation
- Accelerate ray-tracing by providing a hierarchical structure that can be used for bounding volume testing

Ray Casting with Hierarchies



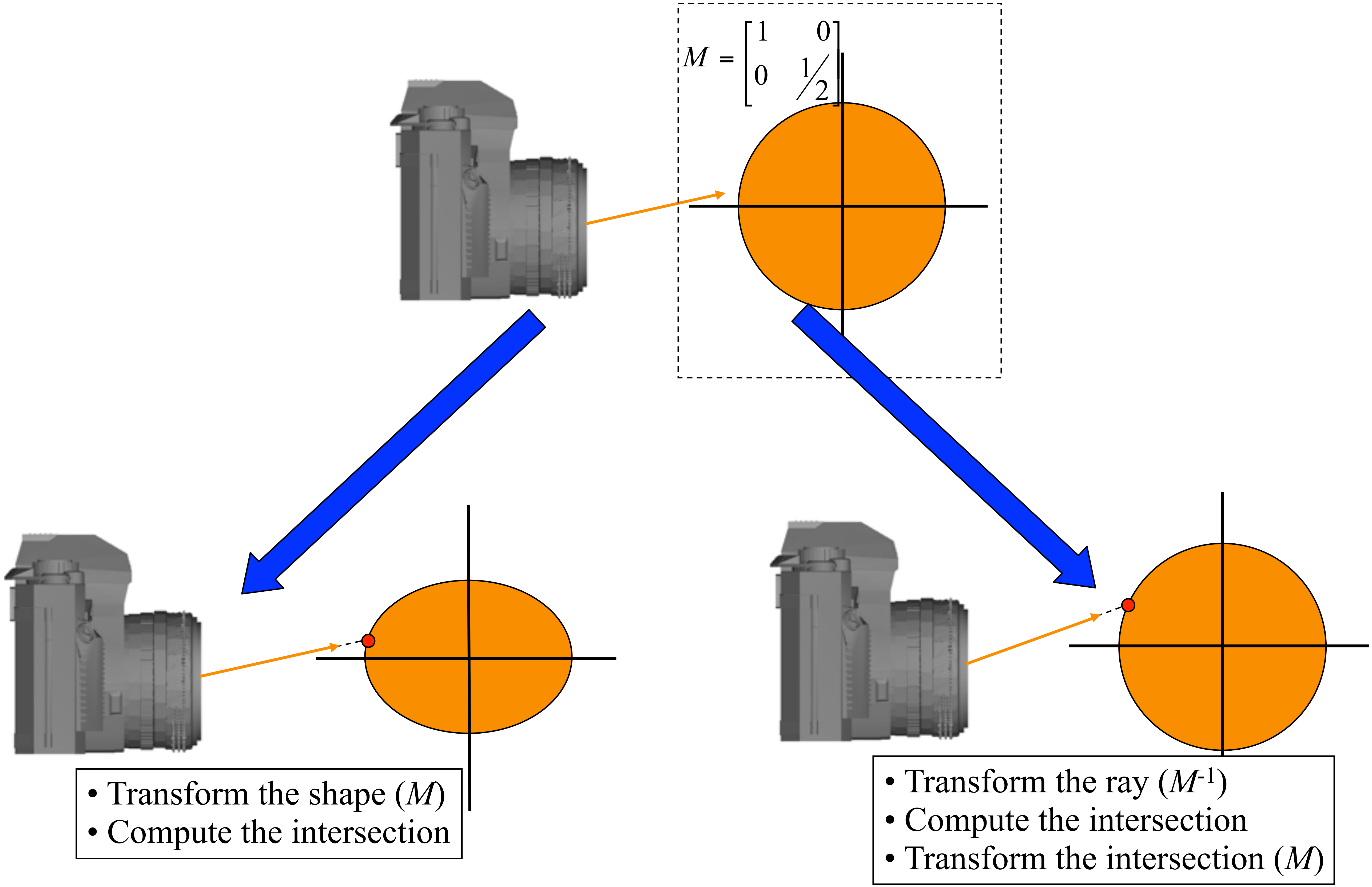
$$M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

Ray Casting with Hierarchies



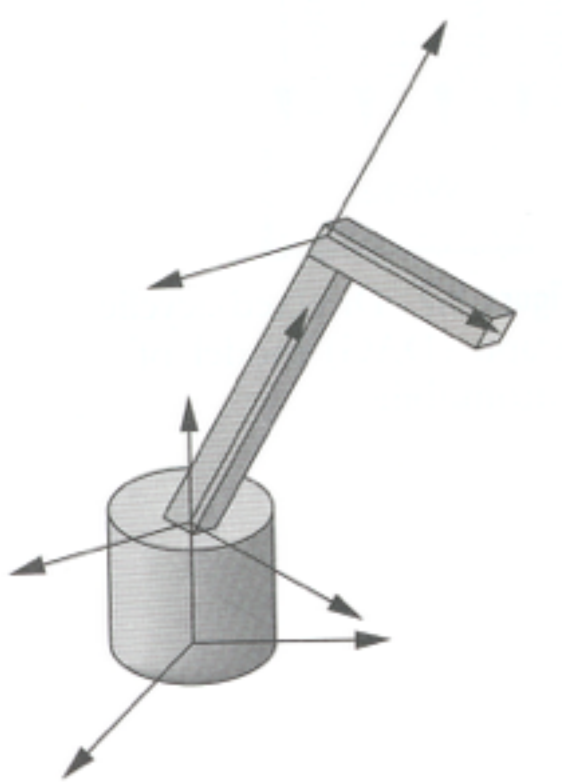
- Transform the shape (M)
- Compute the intersection

Ray Casting with Hierarchies

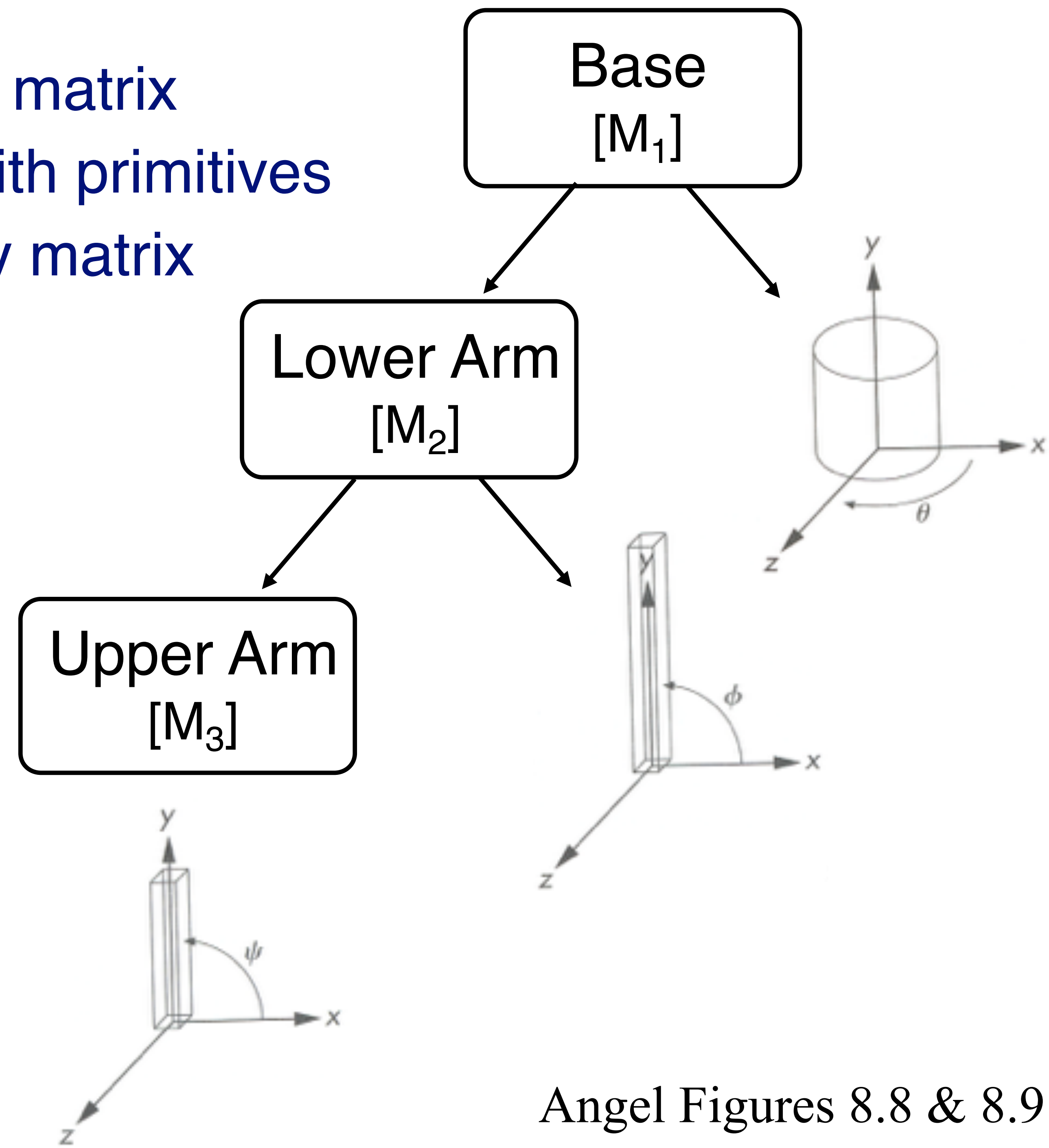


Ray Casting With Hierarchies

- Transform rays, not primitives
  - For each node ...
    - » Transform ray by inverse of matrix
    - » Intersect transformed ray with primitives
    - » Transform hit information by matrix



Robot Arm



Angel Figures 8.8 & 8.9

Applying a Transformation

- Position
- Direction
- Normal

$$\underset{M}{\overset{\text{Affine}}{\begin{bmatrix} a & b & c & tx \\ d & e & f & ty \\ g & h & i & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}}} = \underset{M_T}{\overset{\text{Translate}}{\begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}}} \times \underset{M_L}{\overset{\text{Linear}}{\begin{bmatrix} a & b & c & 0 \\ d & e & f & 0 \\ g & h & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}}$$

Applying a Transformation

- Position
  - Apply the full affine transformation:

    $p' = M(p) = (M_T \times M_L)(p)$

- Direction

- Normal

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Applying a Transformation

- Position
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  - Apply the linear component of the transformation:

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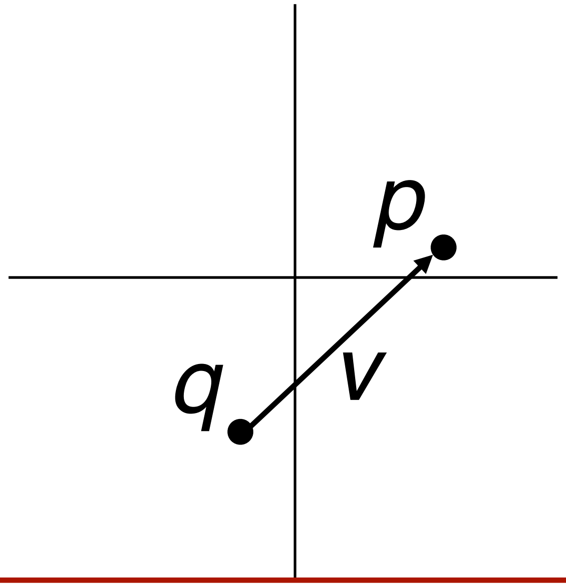
A direction vector *v* is defined as the difference between two positional vectors *p* and *q*: ***v=p-q***.



Applying a Transformation

- Position
- Direction
  - Apply the linear component of the transformation:

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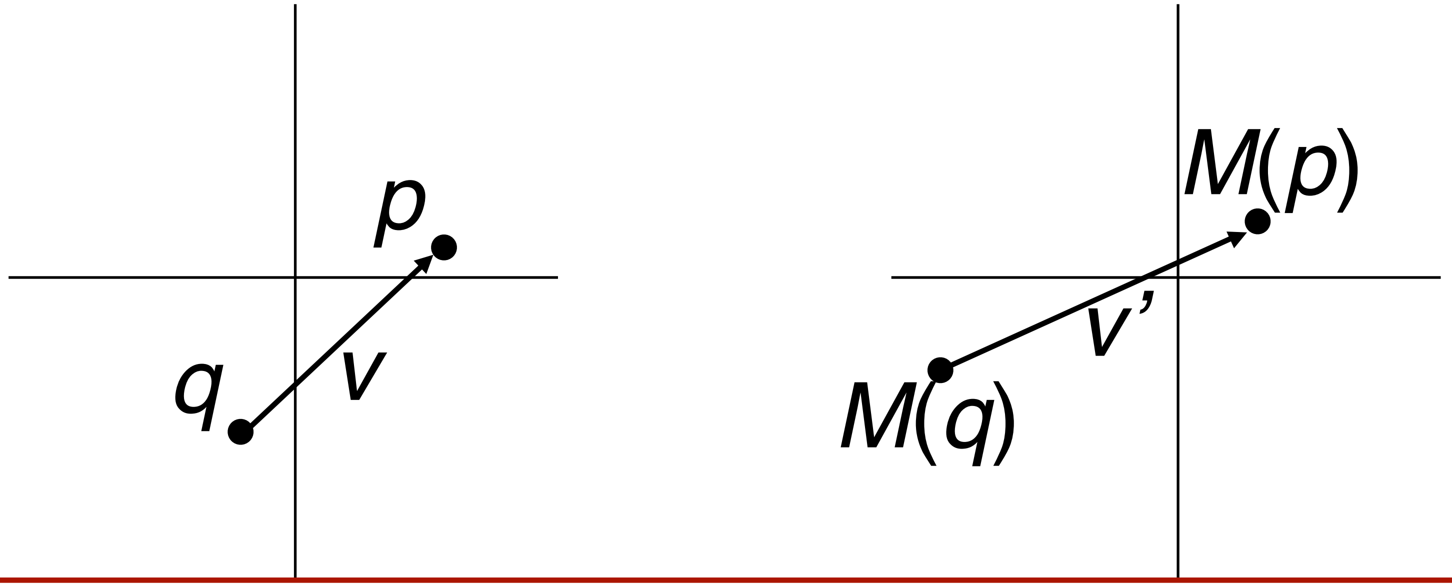
A direction vector *v* is defined as the difference between two positional vectors *p* and *q*: ***v*=*p*-*q***.

Applying the transformation M, we compute the transformed direction as the difference between the transformed positions: ***v'*=M(*p*)-M(*q*)**.



Applying a Transformation

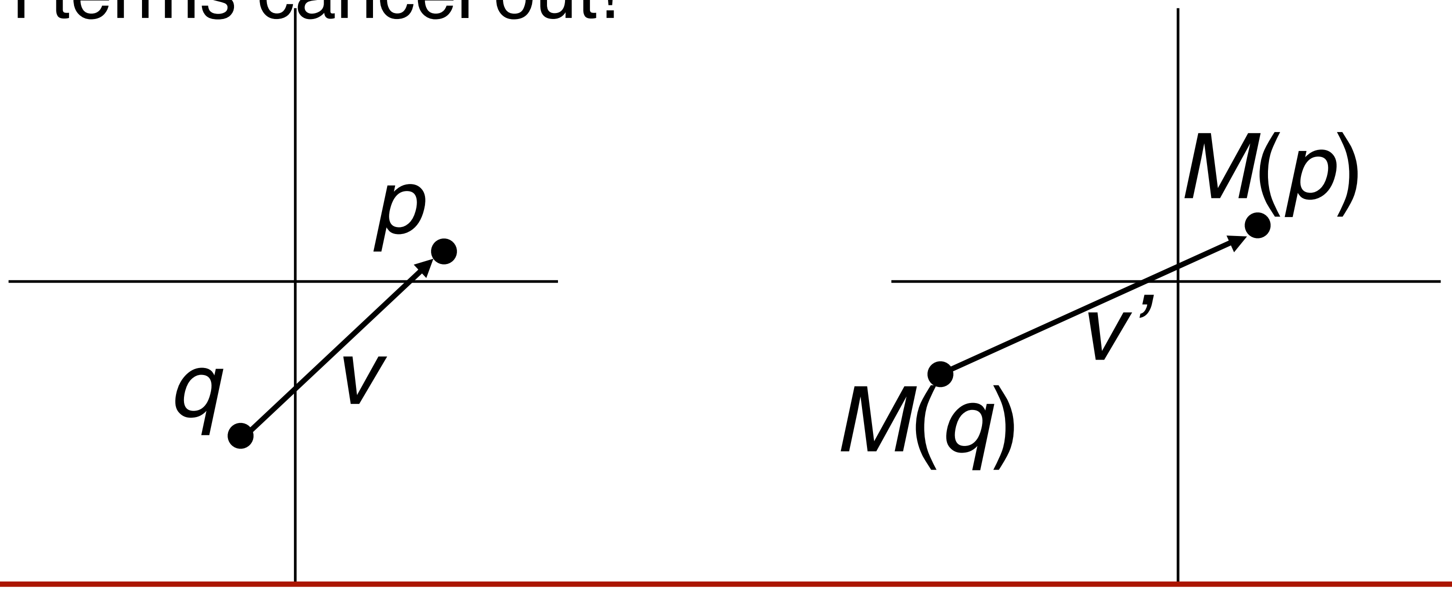
- Position
- Direction
  - Apply the linear component of the transformation:

$$p' = M_L(p)$$

A direction vector v is defined as the difference between two positional vectors p and q: $\boldsymbol{v=p-q}$.

Applying the transformation M, we compute the transformed direction as the difference between the transformed positions: $\boldsymbol{v' = M(p) - M(q)}$.

The translation terms cancel out!



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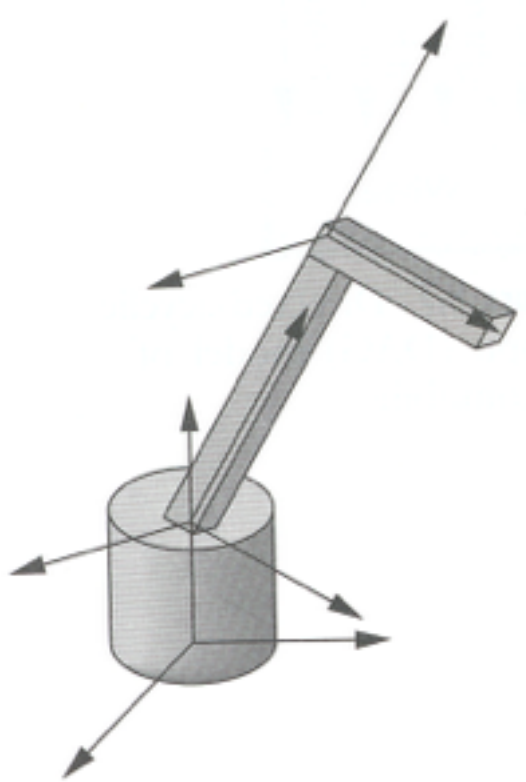

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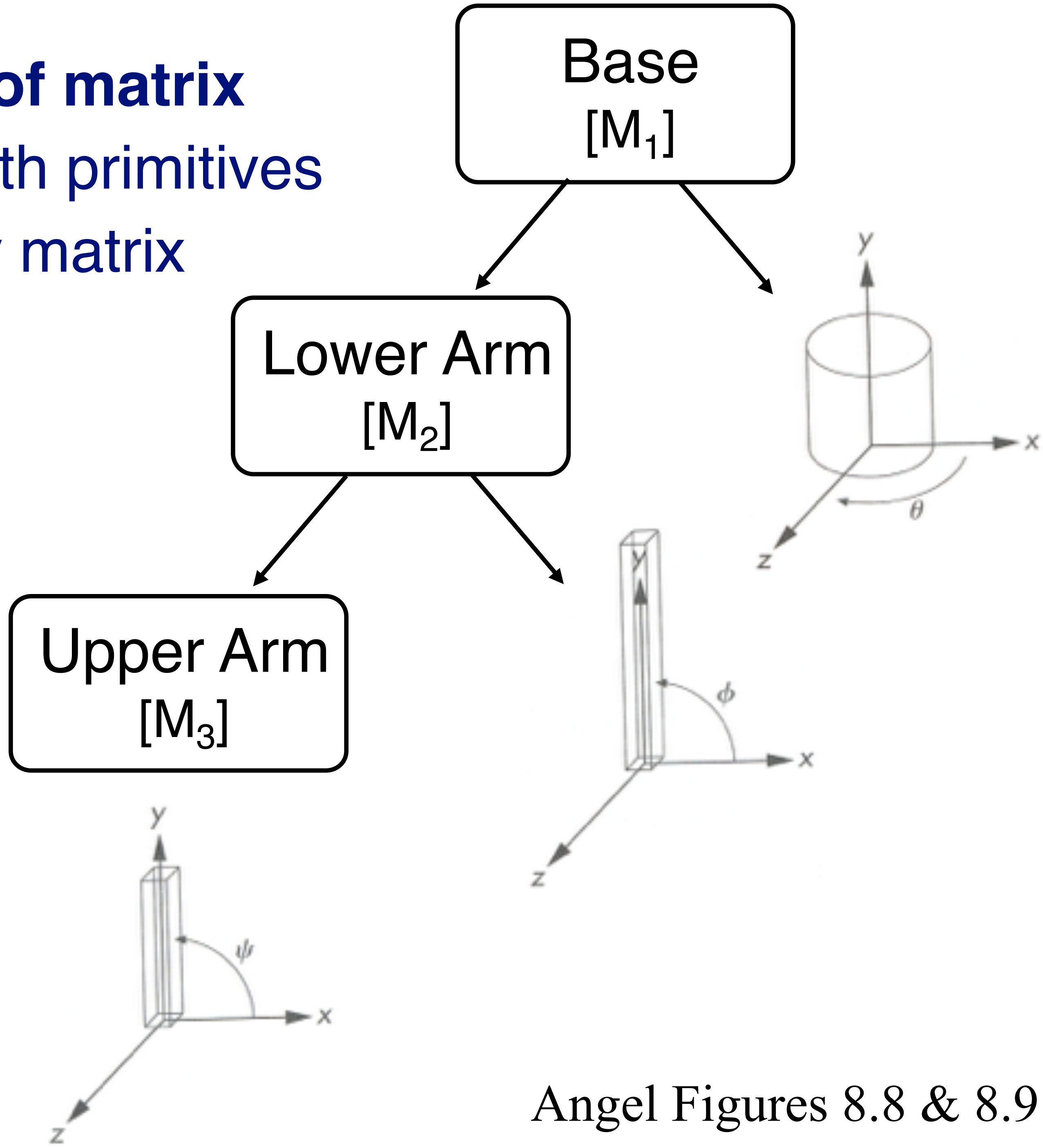

Angel Figures 8.8 & 8.9

Transforming a Ray

- If M is the transformation mapping a scene-graph node into the "world" (or "global") coordinate system, then we transform a ray r by:
  - $r'.\text{start} = M^{-1}(r.\text{start})$
  - $r'.\text{direction} = M_L^{-1}(r.\text{direction})$

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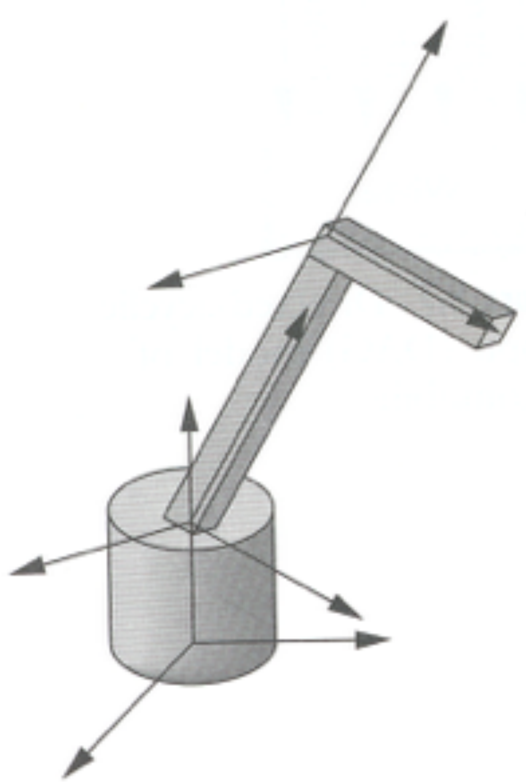

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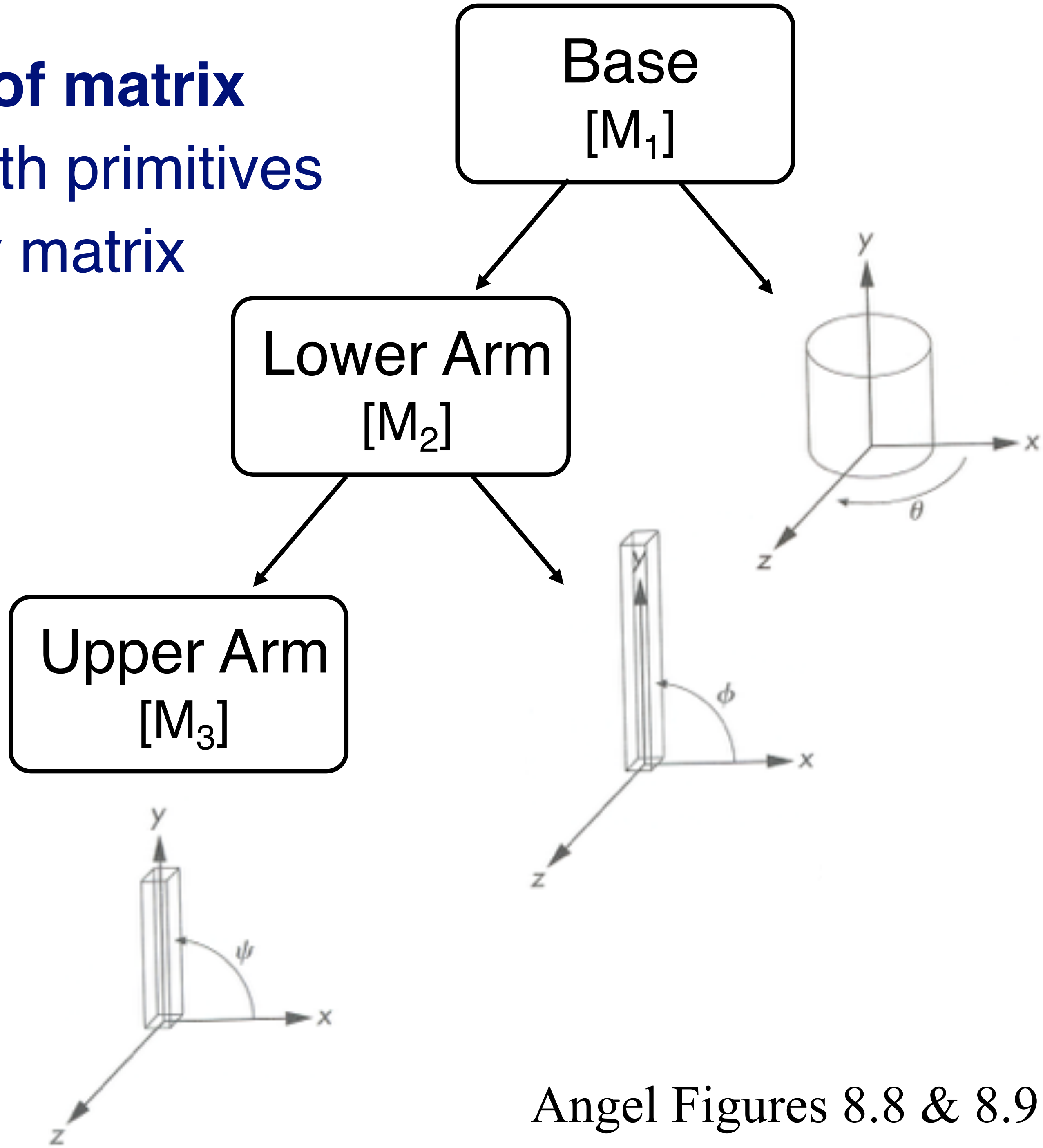

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Applying a Transformation

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$p' = ?$

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Normal Transformation

2D Example:

$$\underset{M}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}} = \underset{M_T}{\overset{\text{Translate}}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}} \times \underset{M_L}{\overset{\text{Scale}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}}$$

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$$\underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}}_{M} = \underbrace{\overset{\text{Translate}}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}}_{M_T} \times \underbrace{\overset{\text{Scale}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}}_{M_L}$$

If *v* is a direction in 2D, and *n* is a vector perpendicular to *v*, we want the transformed *n* to be perpendicular to the transformed *v*:

$$\hat{v} \cdot \hat{n} = 0 \quad \Longrightarrow \quad M_L(\hat{v}) \cdot \hat{n}' = 0$$

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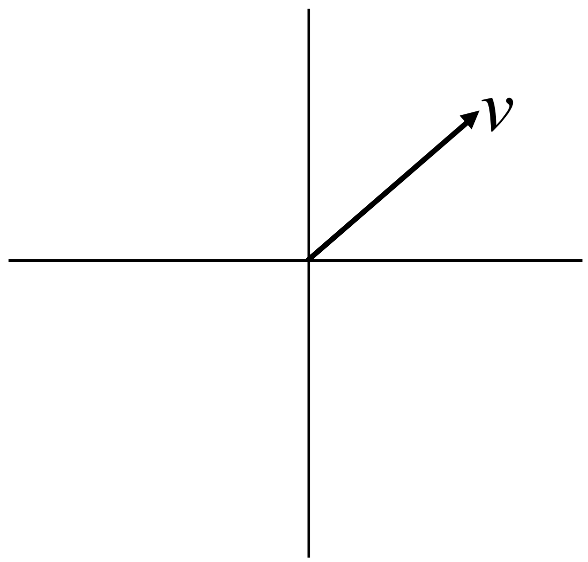
Say $\hat{v} = (2, 2)$ …



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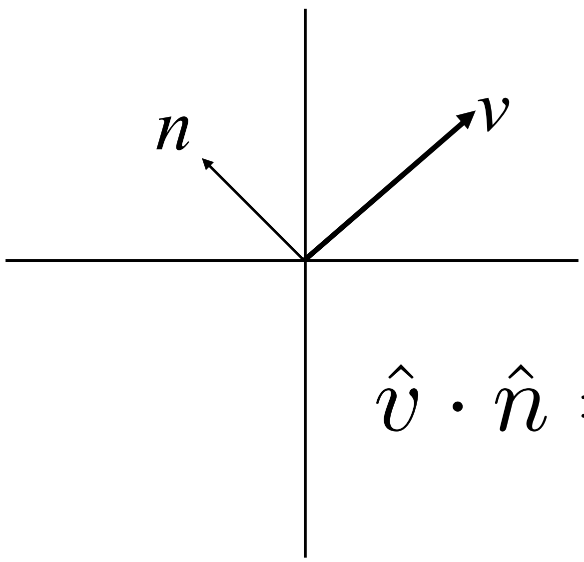
Say $\hat{v} = (2, 2)$ … then $\hat{n} = (-\sqrt{.5}, \sqrt{.5})$



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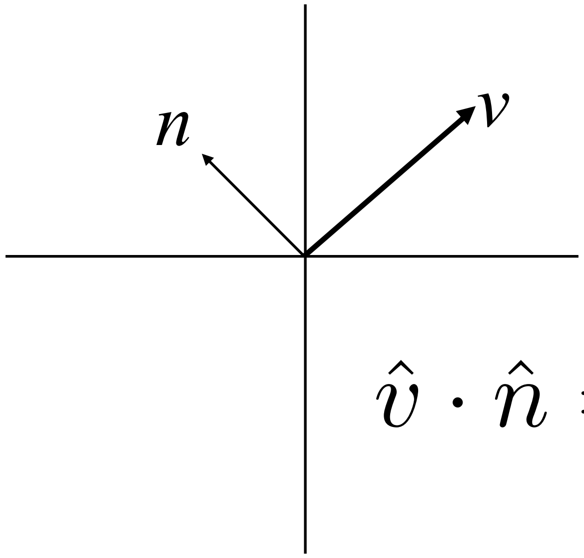
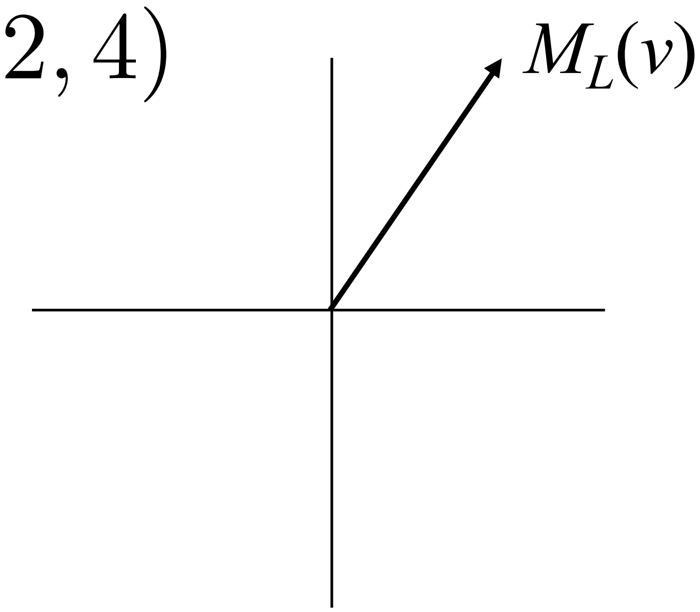


$$\hat{v} \cdot \hat{n} = 0$$

$$M_L(\hat{v}) = (2, 4)$$



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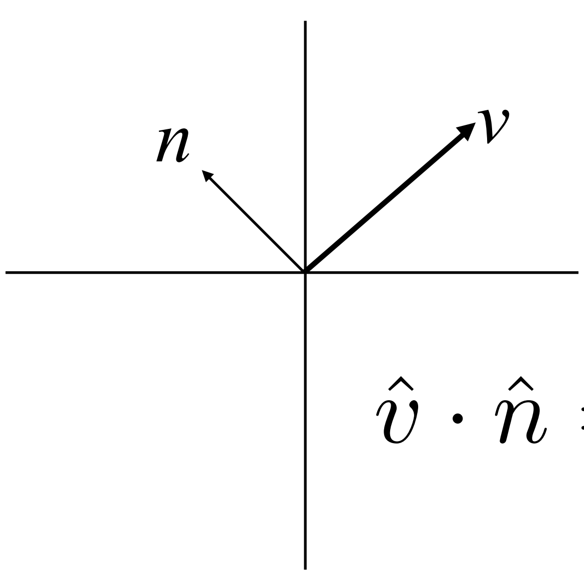

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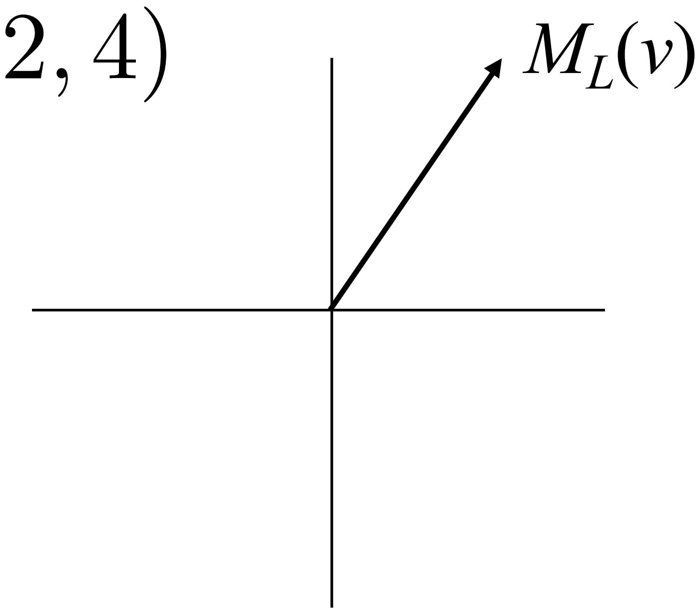

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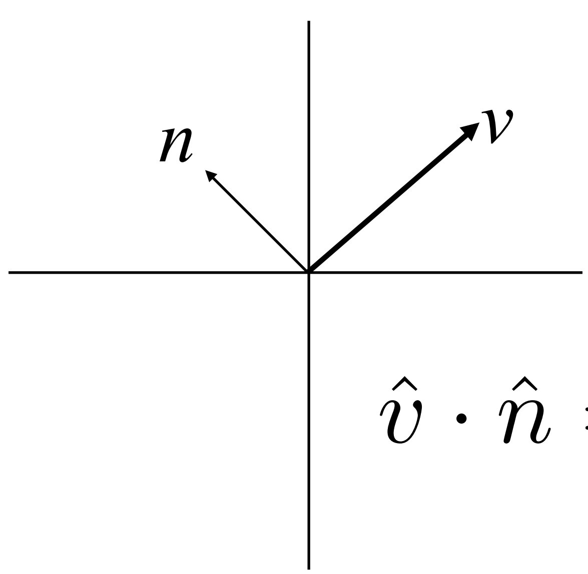

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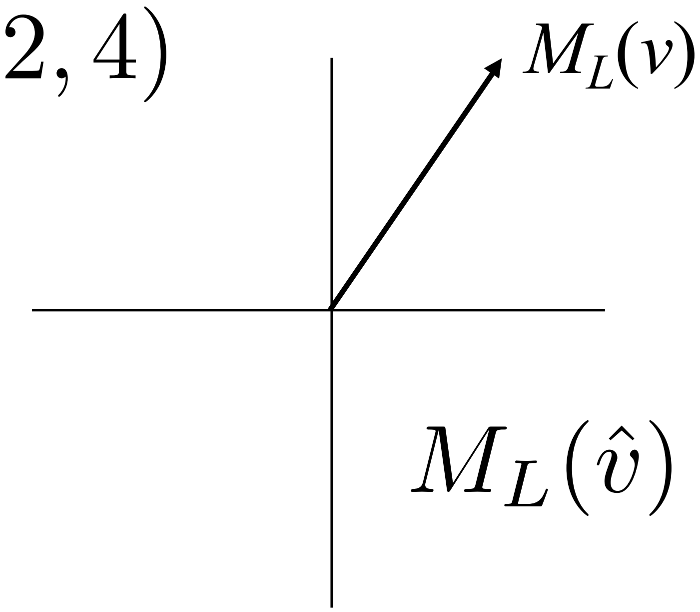

$$M_L(\hat{v}) \cdot M_L(\hat{n}) \neq 0$$

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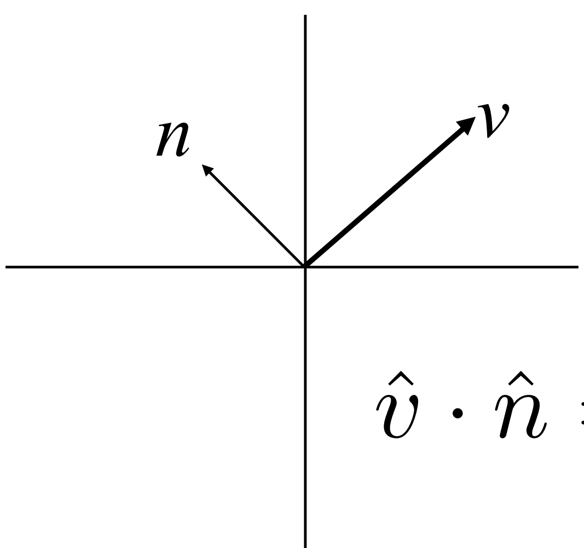
Simply applying the directional part of the transformation to *n* does not result in a vector that is perpendicular to the transformed *v*.

Say $\hat{v} =$ … $-\sqrt{.5}, \sqrt{2})$

$$M_L(\hat{v}) = (2, 4)$$



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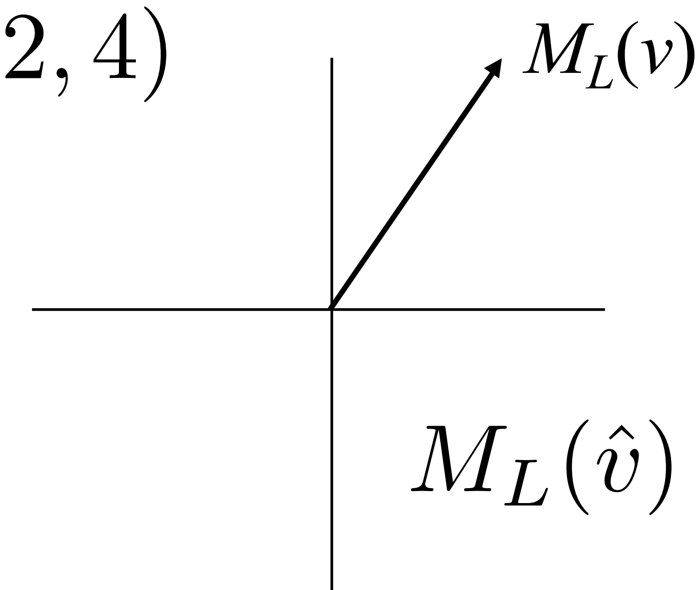

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Recall

Transposes:

- The transpose of a matrix *M* is the matrix *M^t^* whose (i,j)-th coeff. is the (j,i)-th coeff. of M:

$$M = \begin{bmatrix} m_{11} & m_{21} & m_{31} \\ m_{12} & m_{22} & m_{32} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} \qquad M^t = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & 2m_{33} \end{bmatrix}$$

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- If *M* and *N* are two matrices, then the transpose of the product is the inverted product of the transposes:

$$(MN)^t = N^t M^t$$

Recall

Dot-Products :

- The dot product of two vectors $v=(v_x,v_y,v_z)$ and $w=(w_x,w_y,w_z)$ is obtained by summing the product of the coefficients:

$$\hat{v} \cdot \hat{w} = v_x w_x + v_y w_y + v_z w_z$$

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- Can also express as a matrix product:

$$\hat{v} \cdot \hat{w} = \hat{v}^t \hat{w} = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

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Transposes and Dot-Products:

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$$n = M^t n'$$
$$n' = (M^t)^{-1} n$$

Applying a Transformation

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- Direction

  $p' = M_L(p)$

- Normal

  $p' = ((M_L)^t)^{-1}(p)$

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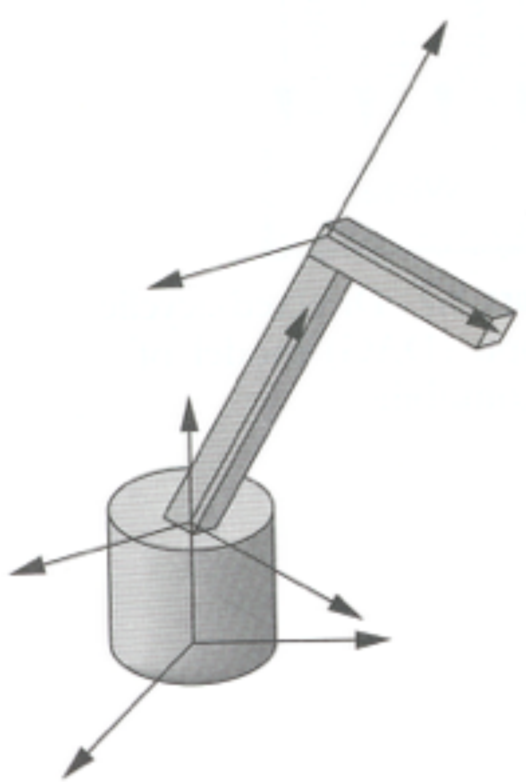
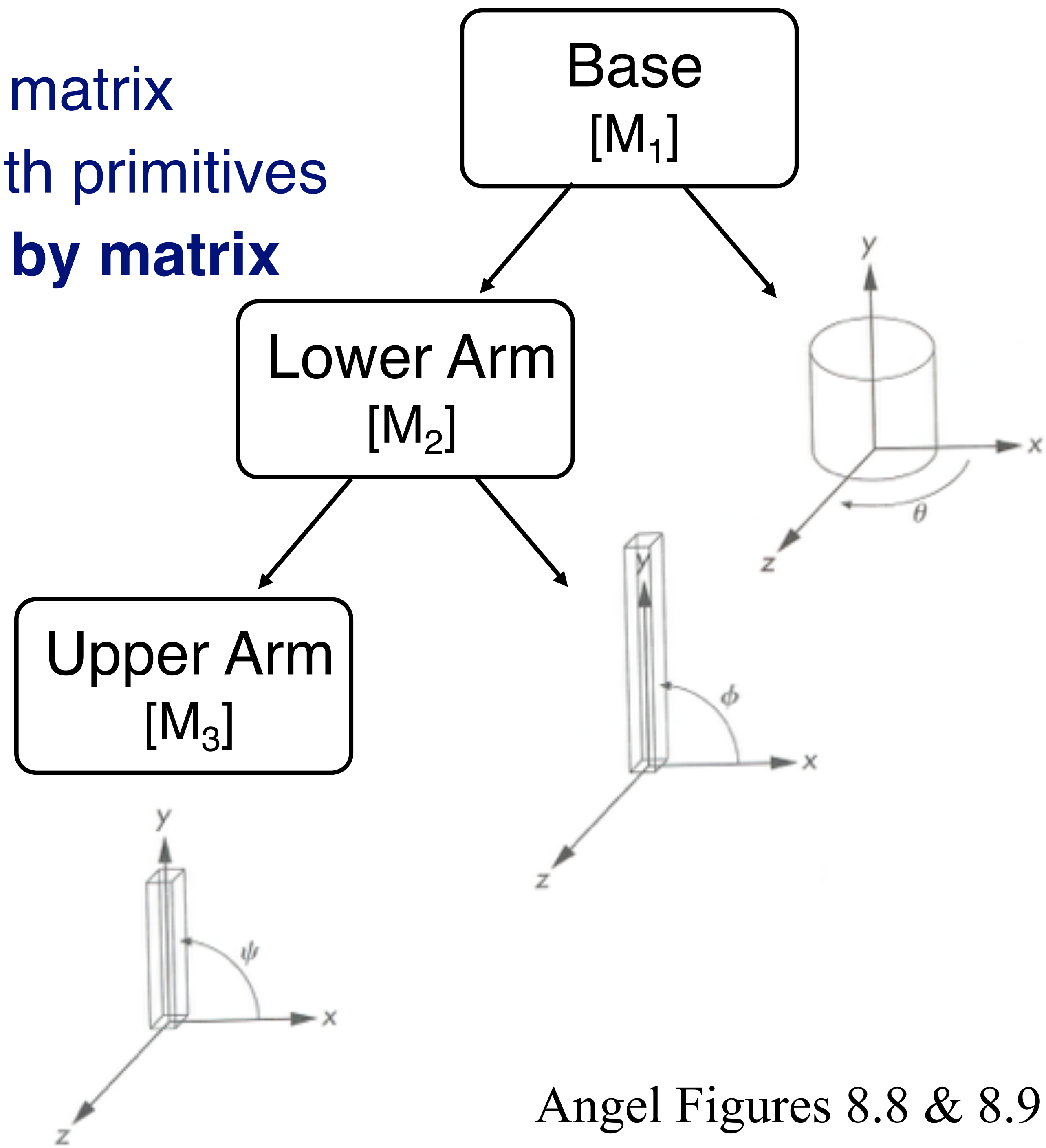
    - » **Transform hit information by matrix**



Robot Arm



Angel Figures 8.8 & 8.9

Transforming a Ray

- If M is the transformation mapping a scene-graph node into the global coordinate system, then we transform the hit information *hit* by:
  - *hit'*.position = M(*hit*.position)
  - *hit'*.normal = $((M_L)^t)^{-1}$(*hit*.normal)

$$\underset{M}{\overset{\text{Affine}}{\begin{bmatrix} a & b & c & tx \\ d & e & f & ty \\ g & h & i & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}}} = \underset{M_T}{\overset{\text{Translate}}{\begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}}} \times \underset{M_L}{\overset{\text{Linear}}{\begin{bmatrix} a & b & c & 0 \\ d & e & f & 0 \\ g & h & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}}$$