## 3D Rendering

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## CS 4810: Graphics

Acknowledgment: slides by Jason Lawrence, Misha Kazhdan, Allison Klein, Tom Funkhouser, Adam Finkelstein and David Dobkin

## Rendering

- Generate an image from geometric primitives


Geometric Primitives


Raster
Image

## Rendering

- Generate an image from geometric primitives


Rendering


3D
2D

## 3D Rendering Example



What issues must be addressed by a 3D rendering system?

## Overview

- 3D scene representation
-3D viewer representation
- Visible surface determination
- Lighting simulation



## Overview

- 3D scene representation
-3D viewer representation
- Visible surface determination
- Lighting simulation

How is the 3D scene described in a computer?


## 3D Scene Representation

- Scene is usually approximated by 3D primitives oPoint
oLine segment
oPolygon
oPolyhedron oCurved surface oSolid object oetc.



## 3D Point

- Specifies a location


## 3D Point

- Specifies a location oRepresented by three coordinates olnfinitely small

```
typedef struct {
    Coordinate x;
    Coordinate y;
    Coordinate z;
} Point;
```

- $(\mathrm{x}, \mathrm{y}, \mathrm{z})$


## 3D Vector

- Specifies a direction and a magnitude



## 3D Vector

- Specifies a direction and a magnitude
oRepresented by three coordinates
oMagnitude IIVII = sqrt(dx dx $+d y d y+d z d z)$
oHas no location


## typedef struct \{

Coordinate dx;
Coordinate dy;
Coordinate dz;
Vector;


## 3D Vector

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```
typedef struct {
Coordinate dx; Coordinate dy;
Coordinate dz;
\} Vector;
```

- Dot product of two 3D vectors

$\mathbf{o V}_{1} \cdot V_{2}=d x_{1} d x_{2}+d y_{1} d y_{2}+d z_{1} d z_{2}$
$o \mathrm{~V}_{1} \cdot \mathrm{~V}_{2}=\left\|\mathrm{V}_{1}\right\|\left\|\mathrm{V}_{2}\right\| \cos (\Theta)$


## Linear Algebra: a Little Review

- What is...?
- $\mathrm{V}_{1} \cdot \mathrm{~V}_{1}=$ ?


## Linear Algebra: a Little Review

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- $V_{1} \cdot V_{1}=d x d x+d y d y+d z d z$


## Linear Algebra: a Little Review

- What is...?
- $\mathrm{V}_{1} \cdot \mathrm{~V}_{1}=(\text { Magnitude })^{2}$


## Linear Algebra: a Little Review

- $\mathrm{V}_{1} \cdot \mathrm{~V}_{1}=(\text { Magnitude })^{2}$
- Now, let $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ both be unit-length vectors.
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## Linear Algebra: a Little Review

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- Now, let $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ both be unit-length vectors.
- What is...?
- $\mathrm{V}_{1} \cdot \mathrm{~V}_{1}=1$
- $\mathrm{V}_{1} \cdot \mathrm{~V}_{2}=$ length of $\mathrm{V}_{1}$ projected onto $\mathrm{V}_{2}$ (or vice-versa)



## 3D Vector

- Specifies a direction and a magnitude
oRepresented by three coordinates
oMagnitude IIVII $=$ sqrt(dx dx $+d y d y+d z d z)$
oHas no location

```
typedef struct {
    Coordinate dx;
    Coordinate dy;
    Coordinate dz;
} Vector;
```

- Cross product of two 3D vectors

$\mathrm{o}_{1} \times \mathrm{V}_{2}=$ Vector normal to plane $\mathrm{V}_{1}, \mathrm{~V}_{2}$
oll $\mathrm{V}_{1} \times \mathrm{V}_{2}\|=\| \mathrm{V}_{1}$ II II $\mathrm{V}_{2} \| \sin (\Theta)$


## Linear Algebra: More Review

- Let $C=A \times B$ :
$o C x=A y B z-A z B y$
$o C y=A z B x-A x B z$
$o C z=A x B y-A y B x$
$-\mathrm{A} \times \mathrm{B}=-\mathrm{B} \times \mathrm{A}$ (remember "right-hand" rule)
- We can do similar derivations to show:
$\mathbf{o} \mathrm{V}_{1} \times \mathrm{V}_{2}=\| \mathrm{V}_{1} I I I I \mathrm{~V}_{2} I I \sin (\Theta) n$, where $n$ is unit vector normal to $V_{1}$ and $V_{2}$
oll $\mathrm{V}_{1} \times \mathrm{V}_{1}$ II $=0$
oll $\mathrm{V}_{1} \times\left(-\mathrm{V}_{1}\right) \|=0$
- http://physics.syr.edu/courses/java-suite/crosspro.html


## 3D Line Segment

- Linear path between two points


## 3D Line Segment

- Use a linear combination of two points oParametric representation:

$$
» P=P_{1}+t\left(P_{2}-P_{1}\right), \quad(0 \leq t \leq 1)
$$

typedef struct \{
Point P1;
Point P2;
\} Segment;


## 3D Ray

- Line segment with one endpoint at infinity oParametric representation:
»P=P1+tV, (0<=t<m)
typedef struct \{
Point P1;
Vector V;
\} Ray;



## 3D Line

- Line segment with both endpoints at infinity oParametric representation:

$$
» P=P_{1}+t V, \quad(-\infty<t<\infty)
$$



## 3D Plane

- A linear combination of three points

$\mathrm{P}_{1}{ }^{\bullet}$


## 3D Plane

- A linear combination of three points
olmplicit representation:

$$
\begin{aligned}
& » P \cdot N+d=0, \text { or } \\
& >a x+b y+c z+d=0
\end{aligned}
$$

```
typedef struct {
    Vector N;
    Distance d;
    Plane;
```



## 3D Polygon

- Area "inside" a sequence of coplanar points
oTriangle
oQuadrilateral
oConvex
oStar-shaped

oConcave
oSelf-intersecting

```
typedef struct {
    Point *points;
    int npoints;
} Polygon;
```

Points are in counter-clockwise order oHoles (use > 1 polygon struct)

## 3D Sphere

- All points at distance "r" from point " $\left(\mathrm{c}_{\mathrm{x}}, \mathrm{c}_{\mathrm{y}}, \mathrm{c}_{\mathrm{z}}\right)$ " olmplicit representation:

$$
»\left(x-c_{x}\right)^{2}+\left(y-c_{y}\right)^{2}+\left(z-c_{z}\right)^{2}=r^{2}
$$

oParametric representation:

$$
\begin{aligned}
& » x=r \cos (\phi) \cos (\Theta)+c_{x} \\
& » y=r \cos (\phi) \sin (\Theta)+c_{y} \\
& » z=r \sin (\phi)+c_{z}
\end{aligned}
$$

Point center;
Distance radius;
\} Sphere;


## Other 3D primitives

- Cone
- Cylinder
- Ellipsoid
- Box
- Etc.


## 3D Geometric Primitives

- More detail on 3D modeling later in course
o Line segment
oPolyhedron
oCurved surface
oSolid object
oetc.


## Overview

- 3D scene representation
-3D viewer representation
- Visible surface determination
- Lighting simulation

How is the viewing device described in a computer?


## Camera Models

- The most common model is pin-hole camera
oAll captured light rays arrive along paths toward focal point without lens distortion (everything is in focus)

View plane

Eye position
(focal point)

## Camera Parameters

-What are the parameters of a camera?


## Camera Parameters

- Position
oEye position (px, py, pz)
- Orientation
oView direction (dx, dy, dz)
oUp direction (ux, uy, uz)
- Aperture
oField of view (xfov, yfov)
- Film plane
o"Look at" point
oView plane normal



## Other Models: Depth of Field



Close Focused


Distance Focused

## Other Models: Motion Blur

- Mimics effect of open camera shutter
- Gives perceptual effect of high-speed motion
- Generally involves temporal super-sampling



## Other Models: Lens Distortion

- Camera lens bends light, especially at edges
- Common types are barrel and pincushion


Barrel Distortion


Pincushion Distortion

## Other Models: Lens Distortion

- Camera lens bends light, especially at edges
- Common types are barrel and pincushion


Barrel Distortion


No Distortion

## Other Models: Lens Distortion

Lens flares are another kind of distortion


Star Wars: Knights of the Old Republic

## Overview

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How can the front-most surface be found with an algorithm?

## Visible Surface Determination

- The color of each pixel on the view plane depends on the radiance emanating from visible surfaces

Simplest method is ray casting


## Ray Casting

- For each sample ...
oConstruct ray from eye position through view plane oFind first surface intersected by ray through pixel oCompute color of sample based on surface radiance



## Ray Casting

- For each sample ...
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## Visible Surface Determination

- For each sample ...
oConstruct ray from eye position through view plane oFind first surface intersected by ray through pixel oCompute color of sample based on surface radiance


More efficient algorithms utilize spatial coherence!

## Rendering Algorithms

Rendering is a problem in sampling and reconstruction!


## Overview

- 3D scene representation
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" Lighting simulation

How do we compute the radiance for each sample ray?


## Lighting Simulation

- Lighting parameters
oLight source emission
oSurface reflectance
oAtmospheric attenuation


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- Lighting parameters
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## Lighting Simulation

- Lighting parameters oLight source emission oSurface reflectance oAtmospheric attenuation

Real-Time Volumetric Shadows paper [Chen et al. 2011]


Durand \& Dorsey Siggraph ‘02

## Lighting Simulation

- Direct illumination
oRay casting
oPolygon shading


More on these methods later!

## Summary

- Major issues in 3D rendering
o3D scene representation
o3D viewer representation
oVisible surface determination
oLighting simulation
- Concluding note
oAccurate physical simulation is complex and intractable
»Rendering algorithms apply many approximations to simplify representations and computations


## Next Lecture

- Ray intersections
- Light and reflectance models
- Indirect illumination

Rendered by Tor Olav Kristensen using POV-Ray


For assignment \#2, you will write a ray tracer!

